ISSN 2581-3463

AJMS

Asian Journal of Mathematical Sciences

RESEARCH ARTICLE

On the Seidel's Method, a Stronger Contraction Fixed Point Iterative Method of Solution for Systems of Linear Equations

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Received: 28-07-2018; Revised: 28-08-2018; Accepted: 29-10-2018

ABSTRACT

In the solution of a system of linear equations, there exist many methods most of which are not fixed point iterative methods. However, this method of Sidel's iteration ensures that the given system of the equation must be contractive after satisfying diagonal dominance. The theory behind this was discussed in sections one and two and the end; the application was extensively discussed in the last section.

Key words: Contraction mapping principle, convergence, program listing, seidel's iterative methods, system linear equations.

INTRODUCTION

This section is concerned with methods for solving the following system of n simultaneous equations in the n unknown $x_1, x_2,\ldots, x_n$:

$$\begin{aligned} f_1(x_1,x_2,\ldots,x_n) &= 0 \\ f_2(x_1,x_2,\ldots,x_n) &= 0 \\ &\vdots \\ f_n(x_1,x_2,\ldots,x_n) &= 0 \end{aligned} \quad (1)$$

However, if these functions are linear in the x's, 1 can be rewritten (Chika[1]) as:

$$\begin{aligned} b_{11}x_1 + b_{12}x_2 + \cdots + b_{1n}x_n &= y_1 \\ b_{12}x_1 + b_{22}x_2 + \cdots + b_{2n}x_n &= y_2 \\ &\vdots \\ b_{n1}x_1 + b_{n2}x_2 + \cdots + b_{nm}x_n &= y_m \end{aligned} \quad (2)$$

More concisely, we (Carnahan[2]) have

$$Bx = y \quad (3)$$

in which B is a matrix of coefficients, $y = (y_1, y_2, y_3, \ldots y_n)$ is the right-hand side vector and $x = (x_1, x_2, x_3, \ldots y_n)$ is the solution vector. Assuming negligible computational round-off error, direct methods considered for this work are the Seidel's method. This iterative technique is more appropriate when dealing with a large number of simultaneous equations (typically of the order of 100 equations or more), which will often possess certain other special characteristics. However, this particular Seidel's iterative method is as in the following theorem.

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Theorem 1 (the main result)

Let $x = f(x)$ be a well-defined map in the metric space (X,ρ) such that

$$x_i = \sum_{i=1}^{n} \alpha_{ij} x_j + \beta \quad (4)$$

Satisfies the Banach's contraction mapping principle then $\bar{x} = \bar{x}_n$ generated by

$$x_{ik} = \sum_{j=1}^{i-1} \alpha_{ij} x_{jk} + \sum \alpha_{ij} x_{jk-1} + \beta \quad (5)$$

for $1 \le i \le n$ and $1 \le k$ when

$$\bar{x}_0 = \bar{x}^0 = x_1^0, x_2^0, \ldots, x_n^0$$

Is the fixed point for 4

Proof

Let x^* be the unique fixed point, then by the contraction principle,

$$x_n = T(x_n)$$

However, $x_1 = T(x_0)$, then

$$\left.\begin{aligned} x_2 &= T(x_1) = T(T(x_0)) = T^2(x_0) \\ x_3 &= T(x_2) = T^2(T(x_0)) = T^3(x_0) \\ &\vdots \\ x_n &= T^{n-1}(T(x_0)) = T^n(x_0) \end{aligned}\right\} \quad (6)$$

Hence, we have constructed a sequence $\{x_n\}_{n=0}$ of linear operators for the Seidel's iterative method defined in the metric space (X,ρ). We now establish that the above-generated sequence is Cauchy. First, we compute $\rho(x_n,x_{n+1}) = \rho(T(x_n,x_{n+1}))$ and by 1.1.3, it is

$$\begin{aligned} &\le KT(x_{n-2}, x_{n-1}) \\ &= K^2 T(x_{n-2}, x_{n-1}) \\ &\vdots \\ &= K^n T(x_0, x_1) \end{aligned}$$

Hence,

$$KT(x_n, x_{n-1}) \le K^n T(x_0, x_1) \quad (7)$$

Now, showing that x_n is Cauchy let $m > n$, then

$$\begin{aligned} \rho(x_n, x_m) &\le \rho(x_n, x_m) + \rho(x_{n-1}, x_{m-1}) + \dots \\ &+ \rho(x_{n-k-1}, x_{m-k-1}) \\ &\le K^n T(x_0, x_1)(1 + K + K^2 + \dots \\ &+ K^{n-m-1} + K^n \end{aligned}$$

Since the series on the right-hand side is a geometric progression with a common ratio < 1, its sum to infinity $\le \frac{1}{1-K}$ so from the above, we have that

$$\rho(x_n, x_m) \le K^n T(x_0, x_1)\left(\frac{1}{1-K}\right) \to 0$$

As $n \to \infty$ since $K < 1$.

Hence, the sequence is Cauchy in (X,ρ) since it is complete and $\{x_n\}$ converges to a point in X.

Let $x_n \to x^*$as $n \to \infty$.

Since T is a contraction and continuous it follows that $T(x_n) \to T(x^*)$ as $n \to \infty$.

However, $T(x_n) = x_{n+1}$, a contraction and continuous it follows that progression with a common ratio number of simultaneous this work is the Seidel

$$x_{n+1} = T(x_n) = T(x^*)$$

Since limits are unique in a metric space and from above, we obtain

$$T(x^*) = x^*$$

We shall now prove that T has a unique fixed point. Suppose for the contraction there exists $y^* \in X$ such that $y^* = x^*$ and $T(x^*) = y^*$

Then

$$\rho(x^*, y^*) = \rho(T(x^*), T(y^*)) \le kT(x^*, y^*)$$

So that

$$(k-1)T(x^*, y^*) \ge 0$$

and

$$T(x^*, y^*) = 0 = \rho$$

We then divide by it to get $k - 1 \ge 0$, i.e., $k \ge 1$ which is a contradiction.

Hence, $x^* = y^*$ and the fixed point is unique. Therefore, $\bar{x} = \bar{x}_n$ generated by Seidel's method

$$x_{ik} = \sum_{j=1}^{i-1} \alpha_{ij} x_{jk} + \sum_{j=i-1}^{n} \alpha_{ij} x_{j,k-1} + \beta$$

Is a fixed point iterative method for the system of equations

$$\sum_{i=1}^{n} \alpha_{ij} x_j + \beta = \sum_{j=1}^{i-1} \alpha_{ij} x_j^{(n-1)}$$

Hence, T has a unique fixed point in (X,ρ)

CONVERGENCE ANALYSIS, AN EXPLANATION TO THE ABOVE MAIN RESULT

To investigate the conditions for the convergence of the Seidel's iterative method, we first phrase the iteration in terms of the individual components. Let x_{ik} denote the *kth* approximation to the *ith* component of the solution vector $x = (x_1,x_2,\dots,\ x_n)^t$. Let $(x_{10},x_{20},\dots,x_{n0})^t$ be an arbitrary initial approximation (though as with the Jacobi method, if a good estimate is known, it should be used for efficiency).[3-6] Let A and v be given and define

$$x_{ik} = \sum_{j=1}^{i-1} a_{ij} x_{jk} + \sum_{j=i+1}^{n} a_{ij} x_{j,k-1} + v_i \qquad (8)$$

For $1 \leq i \leq n$ and $1 \leq k$. When $i = 1, \sum_{j=1}^{i-1} a_{ij} x_{jk}$ is interpreted as zero, and when $i = n, \sum_{j=i+1}^{n} a_{ij} x_{j,k-1}$ is likewise interpreted as zero.

Write A = $A_L + A_R$ where (Eziokwu[6])

$$A_L = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ a_{21} & 0 & \dots & 0 & 0 \\ \vdots & & & \vdots & \\ a_{n1} & a_{n2} & \dots a_{n,n-1} & & 0 \end{bmatrix} \qquad (9)$$

$$A_R = \begin{bmatrix} 0 & a_{12} & \dots & a_{in} \\ 0 & 0 & \dots & a_{2n} \\ \vdots & & & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

Thus, A_L is a strictly lower-triangular matrix whose sub-diagonal entries are the elements of A in their natural positions. A similar description applies to A_R that if $x_k = [x_{1k}, x_{2k}, \dots, x_{nk}]^t$,

$$x_k = A_L x_k + A_R x_{k-1} + v \qquad (10)$$

This can be paraphrased as

$$x_k = (I - A_L)^{-1} A_R x_{k-1} + (I - A_L)^{-1} \qquad (11)$$

which is then of the Jacobi form. This (Chidume[5]) means the necessary and sufficient condition for the convergence is that the eigenvalues of $(I - A_L)^{-1}$ be less in modulus.[7] The eigenvalues of $(I - A_L)^{-1} A_R$ by solving det $((I - A_L)^{-1} A_R - \lambda I) = 0$.

Thus, the Seidel's iterative process converges if all the zeros of the determinant of

$$\begin{bmatrix} -\lambda & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21}\lambda & -\lambda & a_{23} & \dots & a_{2n} \\ a_{31}\lambda & a_{32}\lambda & -\lambda & & a_{3n} \\ \vdots & & & & \vdots \\ a_{n1}\lambda & a_{n2}\lambda & a_{n3}\lambda & \dots & -\lambda \end{bmatrix} \qquad (12)$$

are <1 in absolute value.

Since $a_{ii} = 0, 1 \leq i \leq n$, while $a_{ij} = -b_{ij}/b_i$ the determinant of 12 has the same zero determinants of

$$\begin{bmatrix} b_{11}\lambda & b_{12} & b_{13} & \dots & b_{1n} \\ b_{21}\lambda & b_{22}\lambda & b_{23} & \dots & b_{2n} \\ b_{31}\lambda & b_{32}\lambda & b_{33}\lambda & & b_{3n} \\ \vdots & & & & \vdots \\ b_{n1}\lambda & b_{n2}\lambda & b_{n3}\lambda & \dots & b_{nn}\lambda \end{bmatrix} \qquad (13)$$

It develops that conditions analogous to (9) proved sufficient to guarantee convergence

$$\sum_{\substack{j=1 \\ j\neq i}}^{n} \left| \frac{b_{ij}}{b_{ii}} \right| \leq \mu < 1 \text{ or } \sum_{\substack{j=1 \\ j\neq i}}^{n} \left| \frac{b_{ji}}{b_{jj}} \right| \leq \mu < 1 \qquad (14)$$

The first of these may be demonstrated as previously stated that since

$$|b_{ii}| > \sum_{\substack{j=1 \\ j\neq i}}^{n} |b_{ij}| \qquad (15)$$

B is nonsingular, thus a solution vector x exists and $x = A_x + v$, whence (Argyros[4])

$$x_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ij} x_j + v_i \qquad (16)$$

in which $aij = -b_{ij}/b_{ii}$. Subtracting this yields

$$|x_{ik} - x_i| \leq \sum_{j=1}^{i-1} |a_{ij}| |x_{jk} - x_j| + \sum_{j=i+1}^{n} |a_{ij}| |x_{j,k}| \qquad (17)$$

Let e_k denote the maximum of the numbers $|x_{ik} - x_i|$ as i varies. Then,

$$|x_{ik} - x_i| \leq \sum_{j=2}^{n} |a_{ij}| e_{k-1} \leq \mu e_{k-1} < e_{k-1} \qquad (18)$$

Substituting we (Altman[3]) have

$$|x_{2k} - x_2| \leq |a_{21}| e_{k-1} \leq \mu e_{k-1} \qquad (19)$$

Continuing as indicated gives $|x_{ik} - x_i| \leq \mu e_{k-1}, 1 \leq i \leq n$. This means, of course, that (Friegyes and Nagy[8]) $|xik - x_i| \leq \mu^k e_0$, whence $0 < \mu < 1, \lim_{k\to\infty} x_{ik} = x_i$.

More interesting still than the sufficiency conditions of 18 is the fact that convergence always takes place if the matrix B of 13 is positive definite. To demonstrate this, let $B = L + L + \bar{L}^t$ where $D = \bar{D}$ is the matrix $diag(b_{11}, b_{22}, \ldots, b_{nn})$ and L is the strictly lower-triangular matrix formed from the elements of B below the diagonal. Starting from 14, it is seen that a necessary and sufficient condition for convergence is that all eigenvalues of $(I - A_L)^{-1} A_R$ be of modulus less than unity. However, $A_L = -D^{-L} L$ and $A_R = -D^{-1}L^*$. Thus, $(I - A_L)^{-1} A_R = -(D+L)^{-1} L^*$. The eigenvalues of this matrix, except for sign are those of $(D+L)^{-1} L^*$, which we consider instead. Let λ_i be an eigenvalue of this matrix and let w_i be the corresponding eigenvector. Since B is positive definite,[9]

$$(w_i, Bw_i) = (w_i, Dw_i) + (w_i, Lw_i) + (w_i, L^* w_i) > 0 \tag{20}$$

However, $(D+L)^{-1} L^* w_i = \lambda_i w_i$, so that $L^* w_i = \lambda_i D w_i + \lambda_i L w_i$; then

$$(w_i L^* w_i) = \lambda_i \left[(w_i, Dw_i) + (w_i, Lw_i) \right] \tag{21}$$

Taking the conjugate of both sides,

$(L^* w_i, w_i) = (w_i, Lw_i) = \bar{\lambda}_i \left[(Dw_i, w_i) + (Lw_i, w_i) \right]$,

or

$$(w_i, Lw_i) = \bar{\lambda}_i \left[(w_i, Dw_i) + (w_i, L^* w_i) \right] \tag{22}$$

Combining 21 and 22 gives

$$(w_i, L^* w_i) = \frac{\lambda_i + \lambda_i \bar{\lambda}_i}{I - \lambda_i \bar{\lambda}_i} (w_i, Dw_i),$$

$$(w_i, Lw_i) = \frac{\bar{\lambda}_i + \bar{\lambda}_i \lambda_i}{I - \bar{\lambda}_i \lambda_i} (w_i, Dw_i)$$

Substituting directly in 20 yields

$$\frac{(1+\lambda_i)(1+\bar{\lambda}_i)}{1 - \bar{\lambda}_i \lambda_i} (w_i, Dw_i) > 0$$

Since D is itself positive definite, $(w_i, Dw_i) > 0$; hence, $1 - \bar{\lambda}_i \lambda_i > 0$ or $|\lambda_i| < 1$. Thus, sufficiency has been shown. It is also possible to prove that if the matrix B is Hermitian and all diagonal elements are positive, then convergence requires that B be positive definite.

The solution of systems of equation by iterative procedures such as the Jacobi and Seidel's iterative methods is sometimes termed relaxation (the errors in the initial estimate of the solution vector are decreased or relaxed as calculation continues). The Seidel's iterative method and related methods are used extensively in the solution of large systems of linear equations, generated as the result of the final difference approximation of partial differential equations.

APPLICATION OF SEIDEL'S ITERATIVE METHOD IN THE SOLUTION OF SYSTEMS OF LINEAR EQUATIONS

Problem statement

Write a program that implements the Seidel iterative method described previously for solving the following system of n simultaneous linear equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= a_{1,n+1} \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= a_{2,n+1} \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n &= a_{n,n+1} \end{aligned} \tag{23}$$

in which they a_{ij} are constants.

Method of solution

To reduce the number of divisions required in the calculations, the coefficients of 23 are first normalized by dividing all elements in row i by a_{ii}, $i = 1,2,\ldots,n$, to produce an augmented coefficient matrix of the form[8,10]

$$\begin{bmatrix} 1 & a'_{12} & a'_{13} & \ldots & a'_{1n} & a'_{1,n+1} \\ a'_{21} & 1 & a'_{23} & \ldots & a'_{2n} & a'_{2,n+1} \\ \vdots & & & & \vdots & \vdots \\ a'_{n1} & a'_{n2} & a'_{n3} & \ldots & 1 & a'_{n,n+1} \end{bmatrix} \tag{24}$$

Where, $a'_{ij} = a_{ij}/a_{ii}$

In terms of this notation, the approximation to the solution vector after *kth* iteration,

$$x_k = [x_{1k}, x_{2k}, \ldots, x_{nk}]^t$$

is modified by the algorithm

$$x_{i,k+1} = a'_{i,n+1} - \sum_{j=1}^{i-1} a'_{ij} x_{j,k+1} - \sum_{j=i+1}^{n} a'_{ij} x_{jk}, \quad i = 1,2,\ldots,n \tag{25}$$

to produce the next approximation

$$x_{k+1} = \left[x_{1,k+1}, x_{2,k+1}, \ldots, x_{n,k+1}\right]^t$$

Since, in the Seidel's iterative method the new value $x_{i,k+1}$ replaces the old values x_{ik} as soon as computed the iteration subscript k can be omitted and (25) becomes

$$x_i = a'_{i,n+1} - \sum_{\substack{j=1 \\ j\neq i}}^{n} a'_{ij} x_j, \quad i = 1, 2, \ldots, n \qquad (26)$$

in which the most recently available x_j values are always used on the right-hand side. Hopefully, the x_i values computed by iterating with 26 will converge to the solution of (23).

The convergence criterion is,

$$\left|x_{i,k+1} - x_{ik}\right| < \varepsilon, \quad i = 1, 2, \ldots, n \qquad (27)$$

that is, no element of the solution vector may have its magnitude changed by an amount greater than \varepsilon as a result of one Gauss-Seidel iteration. Since convergence may not occur, an upper limit on the number of iterations. k_{max} is also specified as in the FORTRAN implementation below which flowchart scheme can be seen in the appendix before references.[11-13]

FORTRAN Implementation

Program Symbol	Definition
A	$n \times (n+1)$ augmented coefficient matrix, containing elements a_{ij}
ASTAR, ASTAR	Temporary storage locations for elements of A and X, respectively
EPS	Tolerance used in convergence test, ε
FLAG	A flag used in convergence testing; it has the value 1 for successful convergence and the value 0 otherwise
ITER	Iteration counter, k
ITMAX	The maximum number of iterations allowed k_{max}
N	Number of simultaneous equations, n
X	Vector containing the elements of the current approximation to the solution vector x_k

Program Listing

```
C          APPLIED NUMERICAL METHODS, EXAMPLE 3.3
C          SEIDEL ITERATION FOR N SIMULTANEOUS LINEAR EQUATIONS
C          THE ARRAY A CONTAINS THE N X N + 1 AUGMENTED COEFFICIENT MATRIX
C          THE VECTOR X CONTAINS THE LATEST APPROXIMATION TO THE SOLUTION
C          THE COEFFICIENT MATRIX SHOULD BE DIAGONALLY DOMINANT AND
C          PREFERABLY POSITIVE DEFINITE. ITMAX IS THE MAXIMUM NUMBER OF
C          ITERATIONS ALLOWED. EPS IS USED IN CONVERGENCE TESTING. IN
C          TERMINATING THE ITERATIONS, NO ELEMENT OF X
           MAY UNDERGO A MAGNITUDE
C          CHANGE GREATER THAN EPS FROM ONE ITERATION TO THE NEXT
             INTEGER FLAG
             DIMENSION A (20,20), X (20)
C
C          ......READ AND CHECK INPUT PARAMETERS
C          COEFFICIENT MATRIX AND STARTING VECTOR......
           1        READ (5,100) N, ITMAX, EPS
              WRITE (6,200) N, ITMAX, EPS
              NP1 = N + 1
              READ (5,101) ((A (I, J), J = 1, NP1), I = 1, N)
```

```
          READ (5,101) (X (I), I = 1, N)
          DO 2 I = 1, N
      2        WRITE (6,201) (A (I, J), J = 1, NP1)
          WRITE (6,202) (X (I), I = 1, N)
C
C     ......NORMALIZE DIAGONAL ELEMENTS IN EACH ROW......
      DO 3 I = 1, N
      ASTAR = A (I, I)
          DO 3 J = 1, NP1
      3        A (I, J) = A (I, J)/ASTAR
C
C     ......BEGIN SEIDEL ITERATIONS......
        DO 9 ITER = 1, ITMAX
        FLAG = 1
        DO 7 I = 1, N
        XSTAR = X (I)
        X (I) = A (I, NPI)
C
C     ......FIND NEW SOLUTION VALUE, X (I)......
        DO 5 J = I, N
        IF (I .EQ. J) GO TO 5
        X (I) = X (I) – A (I, J)*X (J)
      5        CONTINUE
C
C     ......TEST X (I) FOR CONVERGENCE......
        IF (ABS (XSTAR – X (I)) .LE.   EPS) GO TO 7
        FLAG = 0
      7        CONTINUE
        IF (FLAG .NE. 1) GO TO 9
          WRITE (6, 203) ITER, (X (I),     I = 1, N)
         GO TO 1
      9        CONTINUE
C     ......REMARK IF METHOD DID NOT CONVERGE......
        WRITE (6,204) ITER, (X (I), I = 1, N)
        GO TO 1
C
C     ......FORMATS FOR INPUT AND OUTPUT STATEMENTS......
      100      FORMAT (6X, 14, 16X, 14, 14X, F10.6)
      101      FORMAT (10X, 6F10.5)
      200      FORMAT (17H1 SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS BY
      GAUSS-SEIDEL METHOD, WITH/1H0, 1 5X, 9HN = 14/
      2 6X, 9HITMAX = , 14/6X, 9HEPS = , F10.5/47H0 THE COEFFICIENT
      3 MATRIX A (1,1).A (N + 1, N + 1) IS)
      201      FORMAT (1H0, 11F10.5)
```

```
202    FORMAT (36 H0 THE STARTING VECTOR
X (1).X (N) IS/(H0, 10F10.5))
203    FORMAT (35H0 PROCEDURE CONVERGED, WITH ITER = , 14/
       1 32H0 SOLUTION VECTOR X (1).X (N) IS/(1H0, 10F10.5))
204    FORMAT (16H0 NO CONVERGENCE/10H0 ITER = , 14/
       1 31H0 CURRENT VECTOR X (1).X (N) IS/(1H0, 10F10.5)) CEND
```

Program Listing (Continued)

Data

N = 4		ITMAX	=	15	EPS	= 0.0001
A (1,1) =	5.0	1.0	3.0	0.0	16.0	1.0
	4.0	1.0	1.0	11.0	-1.0	2.0
	6.0	-2.0	23.0	1.0	-1.0	1.0
	4.0	-2.0				
X (1) =	1.0	2.0	3.0	4.0		
N = 4		ITMAX	=	15	EPS	= 0.0001
A (1,1) =	5.0	1.0	3.0	0.0	16.0	1.0
	4.0	1.0	1.0	11.0	-1.0	2.0
	6.0	-2.0	23.0	1.0	-1.0	1.0
	4.0	-2.0				
X (1) =	50.0	50.0	50.0	50.0		
N = 6		ITMAX	=	50	EPS	= 0.0001
A (1,1) =	4.0	-1.0	0.0	-1.0	0.0	0.0
	100.0	-1.0	4.0	-1.0	0.0	-1.0
	0.0	0.0	0.0	-1.0	4.0	0.0
	0.0	-1.0	0.0	-1.0	0.0	0.0
	4.0	-1.0	0.0	100.0	0.0	-1.0
	0.0	-1.0	4.0	-1.0	0.0	0.0
	0.0	-1.0	0.0	-1.0	4.0	0.0
X (1) =	0.0	0.0	0.0	0.0	0.0	0.0

Computer Output

Results for the 1st Data set

SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS BY SEIDEL'S ITERATIVE METHOD WITH -

N = 4
ITMAX = 15
EPS = 0.00010

THE COEFFICIENT MATRIX A (1,1)...A (N + 1, N + 1) IS

5.00000	1.00000	3.00000	0.0	16.00000
1.00000	4.00000	1.00000	1.00000	11.00000
-1.00000	2.00000	6.00000	-2.00000	23.00000
1.00000	-1.00000	1.00000	4.00000	-2.00000

THE STARTING VECTOR X (1)...X (N) IS

1.00000 2.00000 3.00000 4.00000

PROCEDURE CONVERGED WITH ITER = 12

SOLUTION VECTOR X (1)...X (N) IS

0.99998 2.00000 2.99999 -0.99999

THE STARTING VECTOR X (1)...X (N) IS

50.00000 50.00000 50.00000 50.00000

PROCEDURE CONVERGED WITH ITER = 13

SOLUTION VECTOR X (1)...X (N) IS

1.00002 2.00000 3.00001 -1.00001

Partial Results for the 2nd Data Set (Same Equations as 1st Set)

Results for the 3rd Data Set

SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS BY SEIDEL'S ITERATIVE METHOD, WITH

N = 6

ITMAX = 50

EPS = 0.00010

THE COEFFICIENT MATRIX A (1,1)...A (N + 1, N + 1) IS

4.00000	-1.00000	0.0	-1.00000	0.0	0.0	100.00000
-1.00000	4.00000	-1.00000	0.0	-1.00000	0.0	0.0
0.0	-1.00000	4.00000	0.0	0.0	-1.00000	0.0
-1.00000	0.0	0.0	4.00000	-1.00000	0.0	100.00000
0.0	-1.00000	0.0	-1.00000	4.00000	-1.00000	0.0
0.0	0.0	-1.00000	0.0	-1.00000	4.00000	0.0

THE STARTING VECTOR X (1)...X (N) IS

0.0 0.0 0.0 0.0 0.0 0.0

PROCEDURE CONVERGED WITH ITER = 13

SOLUTION VECTOR X (1).X (N) IS

38.09517 14.28566 4.76188 3.09518 14.28568 4.76189

A simple illustrative example

Use the Seidel's iterative method discussed above to illustrate the solution of the simple system of equations below.

$$10x_1 + x_2 + x_3 = 12$$
$$2x_1 + 10x_2 + x_3 = 13$$
$$2x_1 + 3x_2 + 10x_2 = 15$$

Solution

Since the diagonal dominance is satisfied and for i = 1, we have

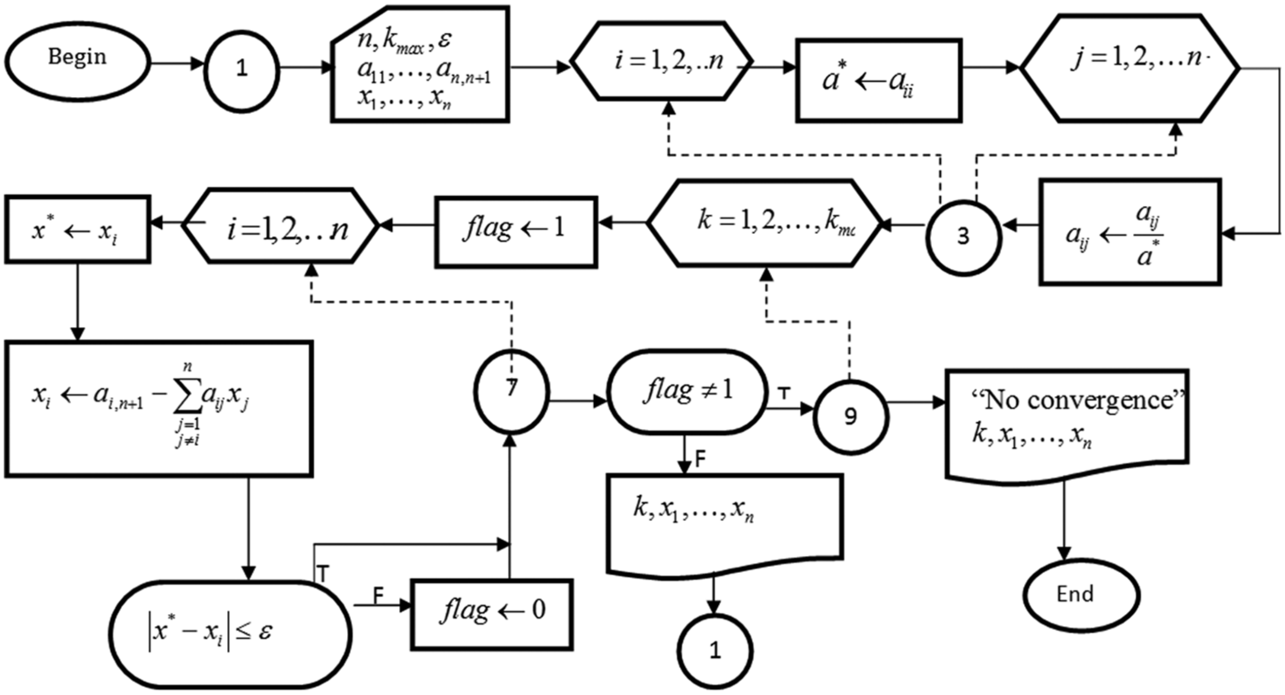
$$x_1^{(i)} = -0.1x_2 - 0.1x_3^{(i-1)} + 1.2$$
$$x_2^{(i)} = -0.2x_1^{(i)} - 0.1x_3^{(i-1)} + 1.3$$
$$x_3^{(i)} = -0.2x_1^{(i)} - 0.3x_2^{(i)} + 1.5$$

with $\bar{x}_0 = (1.2, 1.3, 1.5)$ which gave rise to the table of results below in which $\bar{x}^* = x_{10} = (1,1,1)$ is the fixed point for the given problem in the above example.

	χ_1	χ_2	χ_3
0	1.2	1.3	1.5
1	0.92	0.966	1.1262
2	1.00078	0.997224	1.0006768
3	1.00020992	0.999890336	0.999990915
4	1.000011875	1.000000351	0.999997519
5	1.000000213	1.000000206	0.999999895
6	0.999999989	1.000000013	0.999999998
7	0.999999998	1.000000001	1.000000001
8	0.999999999	0.999999999	1.000000001
9	0.999999999	1.000000000	1.000000000
10	1.000000000	1.000000000	1.000000000

Above table generated on manual solution of parent example above as computed by the corresponding Author, Eziokwu, and test runned using the FORTRAN programming package implementation in page 10 above, under the supervision of the Co-author, Chika

APPENDIX FOR A FLOW DIAGRAM OF THE ABOVE FORTRAN IMPLEMENTATION PROGRAM



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