ISSN 2581-3463

AJMS

Asian Journal of Mathematical Sciences

RESEARCH ARTICLE

Comparison Results of Trapezoidal, Simpson's $\frac{1}{3}$ rule, Simpson's $\frac{3}{8}$ rule, and Weddle's rule

A. Karpagam, V. Vijayalakshmi

Department of Mathematics, SRM Valliammai Engineering College, Chennai, Tamil Nadu, India

Received: 02-08-2018; Revised: 12-09-2018; Accepted: 15-11-2018

ABSTRACT

Numerical integration plays very important role in mathematics. In this paper, overviews on the most common one, namely, trapezoidal Simpson's $\frac{1}{3}$ rule, Simpson's $\frac{3}{8}$ rule and Weddle's rule. Different procedures compared and tried to evaluate the value of some definite integrals. A combined approach of different integral rules has been proposed for a definite integral to get more accurate value for all cases.

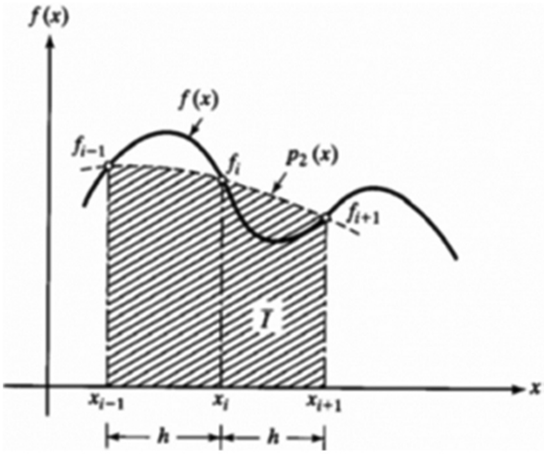
Key words: Numerical integration, Simpson's $\frac{1}{3}$ rule, Simpson's $\frac{3}{8}$ rule, trapezoidal rule, Weddle's rule

INTRODUCTION

Numerical integration of a function of a single variable is called quadrature, which represents the area under the curve $f(x)$ bounded by the ordinates x_0, x_n and X-axis. The numerical integration of a multiple integral is sometimes described as cubature.

Numerical integration problems go back at least to Greek antiquity when, for example, the area of a circle was obtained by successively increasing the number of sides of an inscribed polygon. In the 17th century, the invention of calculus originated a new development of the subject leading to the basic numerical integration rules. In the following centuries, the field became more sophisticated and, with the introduction of computers in the recent past, many classical and new algorithms had been implemented, leading to very fast and accurate results. An extensive research work has already been done by many researchers in the field of numerical integration.[1-5]

Numerical integration $\int_a^b f(x)\,dx$ represents the area between $y=f(x)$, X-axis and the ordinates $x = a$ and $x = b$. This integration is possible only if the $f(x)$ is explicitly given and if it is integrable. If the ordered pairs (x_i, y_i), $i = 0,1,2,..,n$ are plotted and if any two consecutive points are joined by straight line.



The problem of numerical integration can be stated as follows:

Given a set of $(n+1)$ paired values (x_i, y_i), $i = 0,1,2,\ldots,n$ of the function $y = f(x)$ where $f(x)$ is not known explicitly, it is required to compute $\int_{x_0}^{x_n} y\,dx$.

PRELIMINARIES

Definition 2.1

Trapezoidal rule is a technique for approximating the definite integral.[6-7] {\displaystyle\int_{a}^{b}

Address for correspondence:
A.Karpagam,
E-mail: karpagamohan@rediffmail.com

$f(x)$\,dx}The trapezoidal rule works by approximating the region of the function {\displaystyle $f(x)$} $f(x)$ as a trapezoid and calculating its area.{\displays

Definition 2.2

Simpson's rule is a method of numerical integration that provides an approximation of a definite integral over the interval [a,b] using parabolas. The integral of a function *f(x)* over the interval [a,b] subintervals length can be approximated as, as long as n is even.

Definition 2.3

Weddle's rule, a function $f(x)$ be tabulated at points x_i equally spaced by $h = x_{i+1} - x_i$, so $f_1 = f(x_1)$. Then, Weddle's rule approximating the integral of $f(x)$ is given by the Newton–Cotes like formula [Table 1].

Definition 2.4

Quadrature formula for equidistant ordinates
For equally spaced intervals, we have Newton's forward difference formula as follows:

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots$$

Here, $u = \dfrac{x - x_0}{h}$, $x = x_0 + nh$, $dx = h\,du$

$$\int_{x_0}^{x_n} y\,dx = \int_{x_0}^{x_0+nh} f(x)\,dx = \int_{x_0}^{x_0+nh} P_n(x)\,dx$$

$$= \int_0^n \left(y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots \right) h\,du$$

$$= h\int_0^n \left(y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots \right) du$$

$$= h\left[ny_0 + \frac{n^2}{2}\Delta y_0 + \frac{1}{2}\left(\frac{n^3}{3} - \frac{n^2}{2}\right)\Delta^2 y_0 + \ldots\right]$$

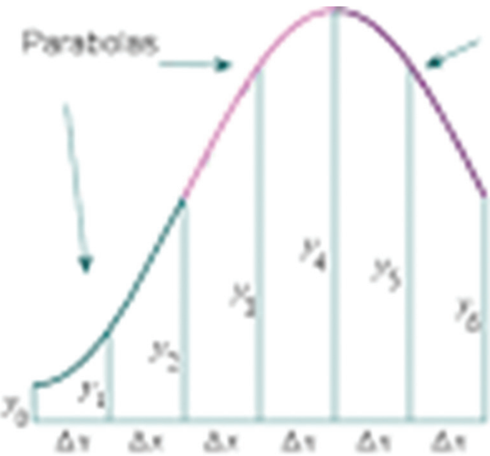
This is called Newton's–Cotes quadrature formula.

Existing rule

Trapezoidal rule is a technique for approximating the definite integral. {\displaystyle\int _{a}^{b} *f(x)*\,*dx*} {\displays
By putting n = 1 in quadrature formula, we get the trapezoidal rule.

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \ldots + y_{n-1})\right]$$

Simpson's rule is a method of numerical integration that provides an approximation of a definite integral over the interval [a,b] using parabolas.
By putting n = 2 in quadrature formula, we get the Simpson's $\frac{1}{3}$ rule.



$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[\begin{array}{l}(y_0 + y_n) + 2(y_2 + y_4 + \ldots + y_{2n}) + \\ 4(y_1 + y_3 + \ldots + \end{array}\right]$$

By putting n = 3 in quadrature formula, we get the Simpson's $\frac{3}{8}$ rule.

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[\begin{array}{l}(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \ldots) + \\ 2(y_3 + y_6 + \ldots + \end{array}\right]$$

Weddle's rule

Weddle's rule, a function $f(x)$ be tabulated at points x_i equally spaced by $h = x_{i+1} - x_i$, so $f_1 = f(x_1)$.[8] Then, Weddle's rule approximating the integral of $f(x)$ is given by the Newton–Cotes like formula

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{10}\left[\begin{array}{l}(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5) + \\ (2y_6 + 5y_7 + y_8 + 6y_9 + y_{10} + 5y_{11}) + \\ (\ldots\ldots\ldots) + (2y_{n-6} + 5y_{n-5} + y_{n-4} + \\ 6y_{n-3} + y_{n-2} + 5y_{n-1} + [y_n) \end{array}\right]$$

Table 1: Comparison results of trapezoidal, Simpson's $\frac{1}{3}$ rule, Simpson's $\frac{3}{8}$ rule, and Weddle's rule

Integral	Actual	Trapezoidal	Simpson's $\frac{1}{3}$	Simpson's $\frac{3}{8}$	Weddle's rule
$\int_{-3}^{3} x^4 dx$	97.2	98.000	97.999	98.000	98.000
	Error	0.8	0.799	0.000	0.000
$\int_{0}^{6} \frac{1}{1+x} dx$	1.9459	2	1.999	2	2
	Error	0.0541	0.0004	0.000	0.000
$\int_{0}^{\pi} \sin x\, dx$	2	1.9825	1.7131	1.5326	1.85746
	Error	0.0175	0.2869	0.4674	0.1425
$\int_{0}^{6} \frac{dx}{1+x^2}$	1.4056	1.083778	1.083759	1.08374	1.08378
	Error	0.321822	0.32186	0.32182	
$\int_{4}^{5.2} \log_e x\, dx$	2.182	2.2804	2.2802	2.2804	2.2803
	Error	0.0984	0.0982	0.0984	0.0981

Proposed rule

Let $y_0, y_1, y_2, \ldots, y_{n-1}, y_n$ be the values of the function $y=f(x)$ at $x_0, x_1, x_2, x_3, x_4, \ldots, x_n$ respectively.

By putting n = 1, h = 1, and sum of all the variables in quadrature formula, we get the trapezoidal rule.

$$\int_{x_0}^{x_n} y dx = \frac{1}{2}[y_0 + y_1 + y_2 + \ldots + y_{n-1} + y_n]$$

The curve on each consecutive pair of intervals is approximated b a parabola. The dotted line is graph of $f(x)$ in [a,b].

By putting n = 2, h = 1 and in quadrature formula, we get the Simpson's $\frac{1}{3}$ rule.

$$\int_{x_0}^{x_n} y dx = \frac{1}{3}[y_0 + y_1 + y_2 + \ldots + y_{n-1} + y_n] + \frac{1}{6}[y_0 + y_1 + y_2 + \ldots + y_{n-1} + y_n]$$

By putting n = 3 and h = 1 in quadrature formula, we get the Simpson's $\frac{3}{8}$ rule.

$$\int_{x_0}^{x_n} y dx = \frac{3}{8} + \frac{1}{8}[y_0 + y_1 + y_2 + \ldots + y_{n-1} + y_n]$$

Weddle's rule

$$\int_{x_0}^{x_n} y dx = \frac{3}{10}[y_0 + y_1 + y_2 + \ldots + y_{n-1} + y_n] + \frac{1}{5}[y_0 + y_1 + y_2 + \ldots + y_{n-1} + y_n]$$

CONCLUSION

In this paper, to find the numerical approximate value of a definite integral $\int_{x_0}^{x_n} f(x)dx$, trapezoidal, Simpson's $\frac{1}{3}$ rule, Simpson's $\frac{3}{8}$ rule, and Weddle's rule are used and its seen that the Weddle's rule gives more accuracy compared the other rules.

REFERENCES

1. Richard LB, Faires JD. Numerical Analysis. Pacific Grove, CA: Wadsworth Group; 2001.

2. Jain MK, Iyengar SR, Jain RK. Numerical Methods for Scientific and Engineering Computation. 6th ed. New Delhi: New Age International; 2012.
3. Frontini M, Sormoni E. Some variants of Newton's method with third order convergence. Appl Math Comput 2003;140:419-26.
4. Jisheng K, Yitian L, Xiuhua W. Third-order modification of Newton's method. J Comput Appl Math 2007; 205:1-5.
5. Jayakumar J, Kalyanasundaram M. Modified Newton's method using harmonic mean for solving nonlinear equations. IOSR J Math 2013;7:93-7.
6. Natarajan A, Mohankumar N. A comparison of some quadrature methods for approximating Cauchy principal value integrals. J Comput Phys 1995;116:365-8.
7. Liu DJ, Wu JM, Yu DH. The super convergence of the Newton-cotes rule for cauchy principal value integrals. J Comput Appl Math 2010;235:696-707.
8. Ullah A. Numerical integration and a proposed rule. Am J Eng Res 2015;4:120-3.