

## RESEARCH ARTICLE

## On Approach to Increase Integration Rate of Elements of an Operational Amplifier Circuit

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**ABSTRACT**

In this paper, we introduce an approach to optimize manufacturing of an operational amplifier circuit based on field-effect transistors. Main aims of the optimization are (i) decreasing dimensions of elements of the considered operational amplifier and (ii) increasing of performance and reliability of the considered field-effect transistors. Dimensions of considered field-effect transistors will be decreased due to manufacture of these transistors framework heterostructure with specific structure, doping of required areas of the heterostructure by diffusion or ion implantation, and optimization of annealing of dopant and/or radiation defects. Performance and reliability of the above field-effect transistors could be increased by optimization of annealing of dopant and/or radiation defects and using inhomogeneity of properties of heterostructure. Choosing of inhomogeneity of properties of heterostructure leads to increasing of compactness of distribution of concentration of dopant. At the same time, one can obtain increasing of homogeneity of the above concentration. In this paper, we also introduce an analytical approach for prognosis of technological process of manufacturing of the considered operational amplifier. The approach gives a possibility to take into account variation of parameters of processes in space and at the same time in space. At the same time, one can take into account nonlinearity of the considered processes.

**Key words:** Integrator operational amplifier, increasing integration rate of field-effect heterotransistors, optimization of manufacturing

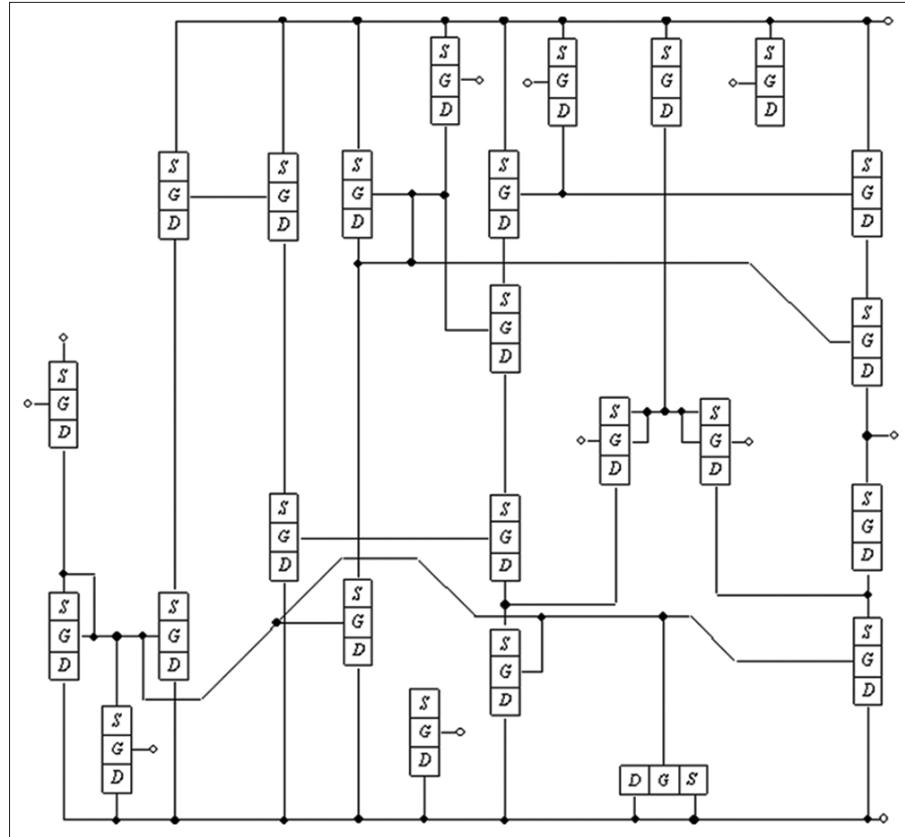
**INTRODUCTION**

An actual and intensively solving problems of solid-state electronics are increasing of integration rate of elements of integrated circuits (*p-n* junctions).<sup>[1-8]</sup> Increasing of the integration rate leads to necessity to decrease their dimensions. To decrease, the dimensions are using several approaches. They are widely using laser and microwave types of annealing of infused dopants. These types of annealing are also widely using for annealing of radiation defects, generated during ion implantation.<sup>[9-17]</sup> Using the approaches leads to generating inhomogeneous distribution of temperature. In this situation several parameters (such as diffusion coefficient and other) became inhomogeneous. Due to the inhomogeneity

one can obtain a possibility to increase integration rate of elements of integrated circuits due to changing of diffusion speed with coordinate. In this situation, one can obtain decreasing dimensions of elements of integrated circuits<sup>[18]</sup> with account Arrhenius law.<sup>[1,3]</sup> Another approach to manufacture elements of integrated circuits with smaller dimensions is doping of heterostructure by diffusion or ion implantation.<sup>[1-3]</sup> However, in this case, optimization of dopant and/or radiation defects is required.<sup>[18]</sup> In this paper, we consider a heterostructure. The heterostructure consists of a substrate and several epitaxial layers. Some sections have been manufactured in the epitaxial layers. Further, we consider doping of these sections by diffusion or ion implantation. The doping gives a possibility to manufacture field-effect transistors framework an integrator circuit so as it is shown in Figure 1. The manufacturing gives a possibility to increase

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**Figure 1:** The considered amplifier circuit<sup>[4]</sup>

density of elements of the operational amplifier circuit.<sup>[4]</sup> After the considered doping, dopant and/or radiation defects should be annealed. Framework the paper, we analyzed the dynamics of redistribution of dopant and/or radiation defects during their annealing. We introduce an approach to decrease the dimensions of the element. However, it is necessary to complicate technological process.

### Method of solution

In this section, we determine spatiotemporal distributions of concentrations of infused and implanted dopants. To determine these distributions, we calculate appropriate solutions of the second Fick's law.<sup>[1,3,18-23]</sup>

$$\begin{aligned} \frac{\partial C(x,y,z,t)}{\partial t} \\ = \frac{\partial}{\partial x} \left[ D_C \frac{\partial C(x,y,z,t)}{\partial x} \right] \\ + \frac{\partial}{\partial y} \left[ D_C \frac{\partial C(x,y,z,t)}{\partial y} \right] \\ + \frac{\partial}{\partial z} \left[ D_C \frac{\partial C(x,y,z,t)}{\partial z} \right] \end{aligned} \quad (1)$$

Boundary and initial conditions for the equations are

$$\begin{aligned} \frac{\partial C(x,y,z,t)}{\partial x} \Big|_{x=0} &= 0, \quad \frac{\partial C(x,y,z,t)}{\partial x} \Big|_{x=L_x} = 0, \\ \frac{\partial C(x,y,z,t)}{\partial y} \Big|_{y=0} &= 0, \quad \frac{\partial C(x,y,z,t)}{\partial y} \Big|_{y=L_y} = 0, \\ \frac{\partial C(x,y,z,t)}{\partial z} \Big|_{z=0} &= 0, \quad \frac{\partial C(x,y,z,t)}{\partial z} \Big|_{z=L_z} = 0, \\ &= 0, \quad C(x,y,z,0) = f(x,y,z) \end{aligned} \quad (2)$$

The function  $C(x,y,z,t)$  describes the spatiotemporal distribution of concentration of dopant;  $T$  is the temperature of annealing;  $D_C$  is the dopant diffusion coefficient. Value of dopant diffusion coefficient could be changed with changing materials of heterostructure, with changing temperature of materials (including annealing), with changing concentrations of dopant and radiation defects. We approximate dependences of dopant diffusion coefficient on parameters by the following relation with account results.<sup>[20-22]</sup>

$$D_C = D_L(x,y,z,T) \left[ 1 + \xi \frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)} \right] \left[ 1 + \xi_1 \frac{V(x,y,z,t)}{V^*} + \xi_2 \frac{V^2(x,y,z,t)}{(V^*)^2} \right] \quad (3)$$

Here, the function  $D_L(x,y,z,T)$  describes the spatial (in heterostructure) and temperature (due to Arrhenius law) dependences of diffusion coefficient of dopant. The function  $P(x,y,z,T)$  describes the limit of solubility of dopant. Parameter  $\gamma \in [1,3]$  describes average quantity of charged defects interacted with atom of dopant.<sup>[20]</sup> The function  $V(x,y,z,t)$  describes the spatiotemporal distribution of the concentration of radiation vacancies. Parameter  $V^*$  describes the equilibrium distribution of concentration of vacancies. The considered concentrational dependence on dopant diffusion coefficient has been described in details<sup>[20]</sup>. It should be noted that using diffusion type of doping did not generate radiation defects. In this situation  $\zeta_1 = \zeta_2 = 0$ , we determine spatiotemporal distributions of concentrations of radiation defects by solving the following system of equations.<sup>[21,22]</sup>

$$\begin{aligned} \frac{\partial I(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[ D_I(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial x} \right] \\ &\quad + \frac{\partial}{\partial y} \left[ D_I(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial y} \right] \\ &\quad + \frac{\partial}{\partial z} \left[ D_I(x,y,z,T) \frac{\partial I(x,y,z,t)}{\partial z} \right] \\ &\quad - k_{I,V}(x,y,z,T) I(x,y,z,t) V(x,y,z,t) \\ &\quad - k_{I,I}(x,y,z,T) I^2(x,y,z,t) \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{\partial V(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[ D_V(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial x} \right] \\ &\quad + \frac{\partial}{\partial y} \left[ D_V(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial y} \right] \\ &\quad + \frac{\partial}{\partial z} \left[ D_V(x,y,z,T) \frac{\partial V(x,y,z,t)}{\partial z} \right] \\ &\quad - k_{I,V}(x,y,z,T) I(x,y,z,t) V(x,y,z,t) \\ &\quad + k_{V,V}(x,y,z,T) V^2(x,y,z,t) \end{aligned}$$

Boundary and initial conditions for these equations are

$$\frac{\partial \rho(x,y,z,t)}{\partial x} \Big|_{x=0} = 0, \quad \frac{\partial \rho(x,y,z,t)}{\partial x} \Big|_{x=L_x} = 0,$$

$$\begin{aligned} \frac{\partial \rho(x,y,z,t)}{\partial y} \Big|_{y=0} &= 0, \quad \frac{\partial \rho(x,y,z,t)}{\partial y} \Big|_{y=L_y} = 0, \\ \frac{\partial \rho(x,y,z,t)}{\partial z} \Big|_{z=0} &= 0, \quad \frac{\partial \rho(x,y,z,t)}{\partial z} \Big|_{z=L_z} = 0, \\ &= 0, \quad r(x,y,z,0) = \text{fr}(x,y,z) \end{aligned} \quad (5)$$

Here,  $\rho = I, V$ . The function  $I(x,y,z,t)$  describes the spatiotemporal distribution of the concentration of radiation interstitials;  $D_\rho(x,y,z,T)$  is the diffusion coefficients of point radiation defects; terms  $V^2(x,y,z,t)$  and  $I^2(x,y,z,t)$  correspond to generation divacancies and di-interstitials;  $k_{I,V}(x,y,z,T)$  is the parameter of recombination of point radiation defects;  $k_{I,I}(x,y,z,T)$  and  $k_{V,V}(x,y,z,T)$  are the parameters of the generation of simplest complexes of point radiation defects. Further, we determine distributions in space and time of concentrations of divacancies  $\Phi_V(x,y,z,t)$  and di-interstitials  $\Phi_I(x,y,z,t)$  by solving the following system of equations.<sup>[21,22]</sup>

$$\begin{aligned} \frac{\partial \Phi_I(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[ D_{\Phi_I}(x,y,z,T) \frac{\partial \Phi_I(x,y,z,t)}{\partial x} \right] \\ &\quad + \frac{\partial}{\partial y} \left[ D_{\Phi_I}(x,y,z,T) \frac{\partial \Phi_I(x,y,z,t)}{\partial y} \right] \\ &\quad + \frac{\partial}{\partial z} \left[ D_{\Phi_I}(x,y,z,T) \frac{\partial \Phi_I(x,y,z,t)}{\partial z} \right] \\ &\quad + k_{I,I}(x,y,z,T) I^2(x,y,z,t) \\ &\quad - k_I(x,y,z,T) I(x,y,z,t) \\ \frac{\partial \Phi_V(x,y,z,t)}{\partial t} &= \frac{\partial}{\partial x} \left[ D_{\Phi_V}(x,y,z,T) \frac{\partial \Phi_V(x,y,z,t)}{\partial x} \right] \\ &\quad + \frac{\partial}{\partial y} \left[ D_{\Phi_V}(x,y,z,T) \frac{\partial \Phi_V(x,y,z,t)}{\partial y} \right] \\ &\quad + \frac{\partial}{\partial z} \left[ D_{\Phi_V}(x,y,z,T) \frac{\partial \Phi_V(x,y,z,t)}{\partial z} \right] \\ &\quad + k_{V,V}(x,y,z,T) V^2(x,y,z,t) \\ &\quad - k_V(x,y,z,T) V(x,y,z,t) \end{aligned} \quad (6)$$

Boundary and initial conditions for these equations are

$$\begin{aligned} \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial x} \right|_{x=0} &= 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \\ \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial y} \right|_{y=0} &= 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial z} \right|_{z=0} &= 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \end{aligned}$$

$$\Phi_I(x, y, z, 0) = f_{\phi_I}(x, y, z), \quad \Phi_V(x, y, z, 0) = f_{\phi_V}(x, y, z) \quad (7)$$

Here,  $D_{\phi\rho}(x, y, z, T)$  is the diffusion coefficients of the above complexes of radiation defects;  $k_I(x, y, z, T)$  and  $k_V(x, y, z, T)$  are the parameters of decay of these complexes.

We calculate distributions of concentrations of point radiation defects in space and time by recently elaborated approach.<sup>[18]</sup> The approach based on transformation of approximations of diffusion coefficients in the following form:  $D_\rho(x, y, z, T) = D_{0\rho}[1 + \varepsilon_\rho g_\rho(x, y, z, T)]$ , where  $D_{0\rho}$  is the average values of diffusion coefficients,  $0 \leq \varepsilon_\rho < 1$ ,  $|g_\rho(x, y, z, T)| \leq 1$ ,  $\rho = I, V$ . We also used analogous transformation of approximations of parameters of recombination of point defects and parameters of generation of their complexes:  $k_{I,V}(x, y, z, T) = k_{0I,V}[1 + \varepsilon_{I,V} g_{I,V}(x, y, z, T)]$ ,  $k_{I,I}(x, y, z, T) = k_{0I,I}[1 + \varepsilon_{I,I} g_{I,I}(x, y, z, T)]$  and  $k_{V,V}(x, y, z, T) = k_{0V,V}[1 + \varepsilon_{V,V} g_{V,V}(x, y, z, T)]$ , where  $k_{0\rho_1,\rho_2}$  is the their average values,  $0 \leq \varepsilon_{I,V} < 1$ ,  $0 \leq \varepsilon_{I,I} < 1$ ,  $0 \leq \varepsilon_{V,V} < 1$ ,  $|g_{I,V}(x, y, z, T)| \leq 1$ ,  $|g_{I,I}(x, y, z, T)| \leq 1$ ,  $|g_{V,V}(x, y, z, T)| \leq 1$ .

Let us introduce the following dimensionless variables:

$$\tilde{I}(x, y, z, t) = I(x, y, z, t)/I^*,$$

$$\tilde{V}(x, y, z, t) = V(x, y, z, t)/V^*,$$

$$\omega = L^2 k_{0I,V} / \sqrt{D_{0I} D_{0V}}, \quad \Omega_\rho = L^2 k_{0\rho,\rho} / \sqrt{D_{0I} D_{0V}},$$

$$\vartheta = \sqrt{D_{0I} D_{0V}} t / L^2, \quad \chi = x / L_x, \quad \eta = y / L_y, \quad \varphi = z / L_z.$$

The introduction leads to transformation of Equations (4) and conditions (5) to the following form:

$$\begin{aligned} &\frac{\partial \tilde{I}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} \\ &= \frac{D_{0I}}{\sqrt{D_{0I} D_{0V}}} \frac{\partial}{\partial \chi} \left\{ \frac{[1 + \varepsilon_I g_I(\chi, \eta, \varphi, T)]}{\partial \tilde{I}(\chi, \eta, \varphi, \vartheta)} \right\} \\ &+ \frac{\partial}{\partial \eta} \left\{ [1 + \varepsilon_I g_I(\chi, \eta, \varphi, T)] \right\} \end{aligned}$$

$$\begin{aligned} &\times \frac{\partial \tilde{I}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \Bigg\} \frac{D_{0I}}{\sqrt{D_{0I} D_{0V}}} + \\ &\frac{D_{0I}}{\sqrt{D_{0I} D_{0V}}} \frac{\partial}{\partial \varphi} \left\{ \frac{[1 + \varepsilon_I g_I(\chi, \eta, \varphi, T)]}{\partial \tilde{I}(\chi, \eta, \varphi, \vartheta)} \right\} \\ &- \tilde{I}(\chi, \eta, \varphi, \vartheta) \\ &\times \omega [1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T)] \tilde{V}(\chi, \eta, \varphi, \vartheta) \\ &- \Omega_I [1 + \varepsilon_{I,I} g_{I,I}(\chi, \eta, \varphi, T)] \tilde{I}^2(\chi, \eta, \varphi, \vartheta) \quad (8) \end{aligned}$$

$$\begin{aligned} &\frac{\partial \tilde{V}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} \\ &= \frac{D_{0V}}{\sqrt{D_{0I} D_{0V}}} \frac{\partial}{\partial \chi} \left\{ \frac{[1 + \varepsilon_V g_V(\chi, \eta, \varphi, T)]}{\partial \tilde{V}(\chi, \eta, \varphi, \vartheta)} \right\} \\ &+ \frac{\partial}{\partial \eta} \left\{ [1 + \varepsilon_V g_V(\chi, \eta, \varphi, T)] \right\} \\ &\times \frac{\partial \tilde{V}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \Bigg\} \frac{D_{0V}}{\sqrt{D_{0I} D_{0V}}} \\ &+ \frac{D_{0V}}{\sqrt{D_{0I} D_{0V}}} \frac{\partial}{\partial \varphi} \left\{ \frac{[1 + \varepsilon_V g_V(\chi, \eta, \varphi, T)]}{\partial \tilde{V}(\chi, \eta, \varphi, \vartheta)} \right\} \\ &- \tilde{I}(\chi, \eta, \varphi, \vartheta) \\ &\times \omega [1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T)] \tilde{V}(\chi, \eta, \varphi, \vartheta) \\ &- \Omega_V [1 + \varepsilon_{V,V} g_{V,V}(\chi, \eta, \varphi, T)] \tilde{V}^2(\chi, \eta, \varphi, \vartheta) \\ &\frac{\partial \tilde{\rho}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \Bigg|_{\chi=0} = 0, \quad \frac{\partial \tilde{\rho}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \Bigg|_{\chi=1} = 0, \\ &\frac{\partial \tilde{\rho}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \Bigg|_{\eta=0} = 0, \quad \frac{\partial \tilde{\rho}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \Bigg|_{\eta=1} = 0, \\ &\frac{\partial \tilde{\rho}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \Bigg|_{\varphi=0} = 0, \quad \frac{\partial \tilde{\rho}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \Bigg|_{\varphi=1} = 0, \\ &\tilde{\rho}(\chi, \eta, \varphi, \vartheta) = \frac{f_\rho(\chi, \eta, \varphi, \vartheta)}{\rho^*} \quad (9) \end{aligned}$$

We determine solutions of Equations (8) with conditions (9) framework recently introduced approach,<sup>[18]</sup> i.e., as the power series

$$\tilde{\rho}(\chi, \eta, \varphi, \vartheta) = \sum_{i=0}^{\infty} \varepsilon_{\rho}^i \sum_{j=0}^{\infty} \omega^j \sum_{k=0}^{\infty} \Omega_{\rho}^k \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta) \quad (10)$$

Substitution of the series (10) into Equations (8) and conditions (9) gives us possibility to obtain equations for initial order approximations of concentration of point defects  $\tilde{I}_{000}(\chi, \eta, \varphi, \vartheta)$  and  $\tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)$  and corrections for them

$$\tilde{I}_{ijk}(\chi, \eta, \varphi, \vartheta) \text{ and } \tilde{V}_{ijk}(\chi, \eta, \varphi, \vartheta), \quad i \geq 1, j \geq 1,$$

$k \geq 1$ . The equations are presented in the Appendix. Solutions of the equations could be obtained by standard Fourier approach.<sup>[24,25]</sup> The solutions are presented in the Appendix.

Now, we calculate distributions of concentrations of simplest complexes of point radiation defects in space and time. To determine the distributions, we transform approximations of diffusion coefficients in the following form:  $D_{\Phi\rho}(x, y, z, T) = D_{0\Phi\rho}[1 + \varepsilon_{\Phi\rho}g_{\Phi\rho}(x, y, z, T)]$ , where  $D_{0\Phi\rho}$  is the average values of diffusion coefficients. In this situation, the Equation (6) could be written as

$$\begin{aligned} \frac{\partial \Phi_I(x, y, z, t)}{\partial t} &= \\ D_{0\Phi I} \frac{\partial}{\partial x} \left\{ \frac{\left[1 + \varepsilon_{\Phi I}g_{\Phi I}(x, y, z, T)\right]}{\partial \Phi_I(x, y, z, t)} \right\} & \\ + k_{I,I}(x, y, z, T) I^2(x, y, z, t) + & \\ + D_{0\Phi I} \frac{\partial}{\partial y} \left\{ \frac{\left[1 + \varepsilon_{\Phi I}g_{\Phi I}(x, y, z, T)\right]}{\partial \Phi_I(x, y, z, t)} \right\} & \\ + D_{0\Phi I} \frac{\partial}{\partial z} \left\{ \frac{\left[1 + \varepsilon_{\Phi I}g_{\Phi I}(x, y, z, T)\right]}{\partial \Phi_I(x, y, z, t)} \right\} & \\ - k_I(x, y, z, T) I(x, y, z, t) & \\ \frac{\partial \Phi_V(x, y, z, t)}{\partial t} &= \\ = D_{0\Phi V} \frac{\partial}{\partial x} \left\{ \frac{\left[1 + \varepsilon_{\Phi V}g_{\Phi V}(x, y, z, T)\right]}{\partial \Phi_V(x, y, z, t)} \right\} & \\ + k_{I,I}(x, y, z, T) I^2(x, y, z, t) & \end{aligned}$$

$$\begin{aligned} + D_{0\Phi V} \frac{\partial}{\partial y} \left\{ \frac{\left[1 + \varepsilon_{\Phi V}g_{\Phi V}(x, y, z, T)\right]}{\partial \Phi_V(x, y, z, t)} \right\} & \\ + D_{0\Phi V} \frac{\partial}{\partial z} \left\{ \frac{\left[1 + \varepsilon_{\Phi V}g_{\Phi V}(x, y, z, T)\right]}{\partial \Phi_V(x, y, z, t)} \right\} & \\ - k_I(x, y, z, T) I(x, y, z, t). & \end{aligned}$$

Farther, we determine solutions of above equations as the following power series:

$$\Phi_{\rho}(x, y, z, t) = \sum_{i=0}^{\infty} \varepsilon_{\Phi\rho}^i \Phi_{\rho i}(x, y, z, t) \quad (11)$$

Now, we used the series (11) into Equation (6) and appropriate boundary and initial conditions. The using gives the possibility to obtain equations for initial order approximations of concentrations of complexes of defects  $\Phi\rho_0(x, y, z, t)$ , corrections for them  $\Phi\rho_i(x, y, z, t)$  (for them  $i \geq 1$ ), and boundary and initial conditions for them. We remove equations and conditions to the Appendix. Solutions of the equations have been calculated by standard approaches<sup>[24,25]</sup> and presented in the Appendix.

Now, we calculate distribution of concentration of dopant in space and time using the approach, which was used for analysis of radiation defects. To use the approach, we consider the following transformation of approximation of dopant diffusion coefficient:  $D_L(x, y, z, T) = D_{0L}[1 + \varepsilon_L g_L(x, y, z, T)]$ , where  $D_{0L}$  is the average value of dopant diffusion coefficient,  $0 \leq \varepsilon_L < 1$ ,  $|g_L(x, y, z, T)| \leq 1$ . Farther, we consider solution of Equation (1) as the following series:

$$C(x, y, z, t) = \sum_{i=0}^{\infty} \varepsilon_L^i \sum_{j=1}^{\infty} \xi_j^i C_{ij}(x, y, z, t).$$

Using the relation into Equation (1) and conditions (2) leads to obtain equations for the functions  $C_{ij}(x, y, z, t)$  ( $i \geq 1, j \geq 1$ ), boundary and initial conditions for them. The equations are presented in the Appendix. Solutions of the equations have been calculated by standard approaches.<sup>[24,25]</sup> The solutions are presented in the Appendix.

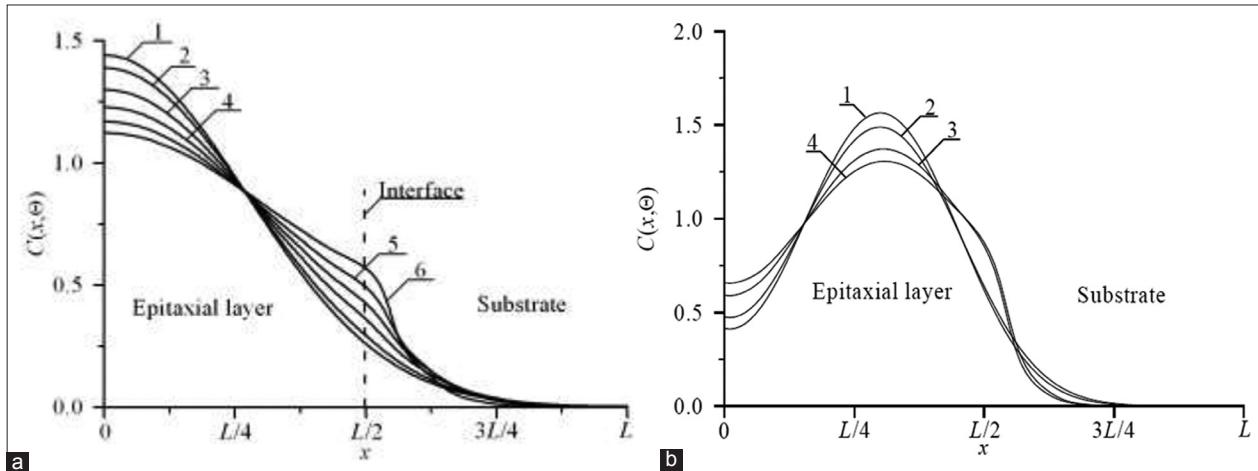
We analyzed distributions of concentrations of dopant and radiation defects in space and time analytically using the second-order approximations on all parameters, which have been used in appropriate series. Usually, the second-order approximations are enough good approximations to make qualitative analysis and

to obtain quantitative results. All analytical results have been checked by numerical simulation.

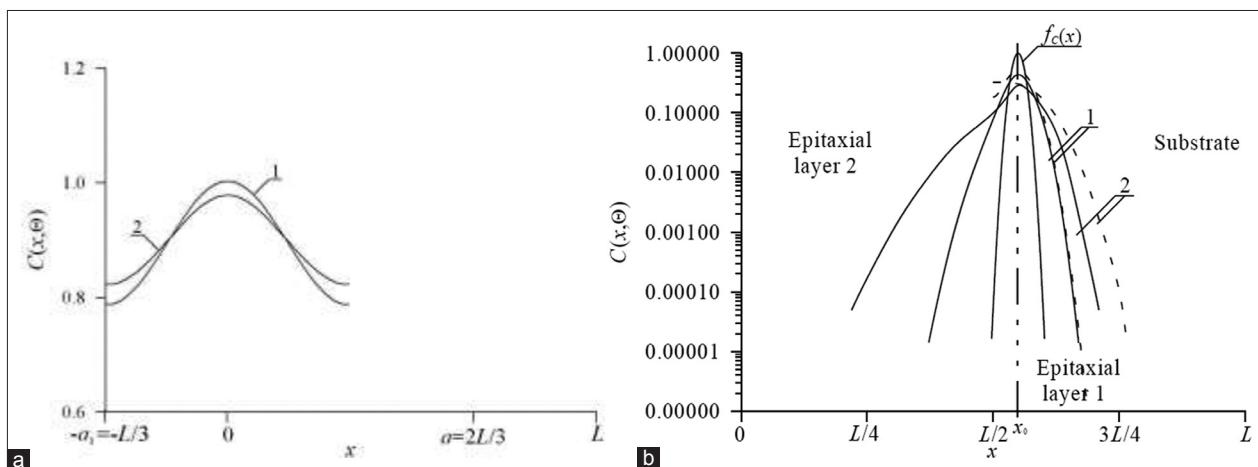
## DISCUSSION

In this section, we analyzed spatiotemporal distributions of concentrations of dopants. Figure 2 shows typical spatial distributions of concentrations of dopants in neighborhood of interfaces of heterostructures. We calculate these distributions

of concentrations of dopants under the following condition: Value of dopant diffusion coefficient in doped area is larger than the value of dopant diffusion coefficient in nearest areas. In this situation, one can find increasing of compactness of field-effect transistors with increasing of homogeneity of distribution of concentration of dopant at one time. Changing relation between values of dopant diffusion coefficients leads to opposite result [Figure 3]. It should be noted that framework the considered



**Figure 2:** (a) Dependences of concentration of dopant, infused in heterostructure from Figure 1, on coordinate in direction, which is perpendicular to interface between epitaxial layer substrate. Difference between values of dopant diffusion coefficient in layers of heterostructure increases with increasing of number of curves. Value of dopant diffusion coefficient in the epitaxial layer is larger than the value of dopant diffusion coefficient in the substrate. (b) Dependences of concentration of dopant, implanted in heterostructure from Figure 1, on coordinate in direction, which is perpendicular to interface between epitaxial layer substrate. Difference between values of dopant diffusion coefficient in layers of heterostructure increases with increasing of number of curves. Value of dopant diffusion coefficient in the epitaxial layer is larger than the value of dopant diffusion coefficient in the substrate. Curve 1 corresponds to homogeneous sample and annealing time  $\Theta = 0.0048 (L_x^2 + L_y^2 + L_z^2)/D_0$ . Curve 2 corresponds to homogeneous sample and annealing time  $\Theta = 0.0057 (L_x^2 + L_y^2 + L_z^2)/D_0$ . Curves 3 and 4 correspond to heterostructure from Figure 1; annealing times  $\Theta = 0.0048 (L_x^2 + L_y^2 + L_z^2)/D_0$  and  $\Theta = 0.0057 (L_x^2 + L_y^2 + L_z^2)/D_0$ , respectively

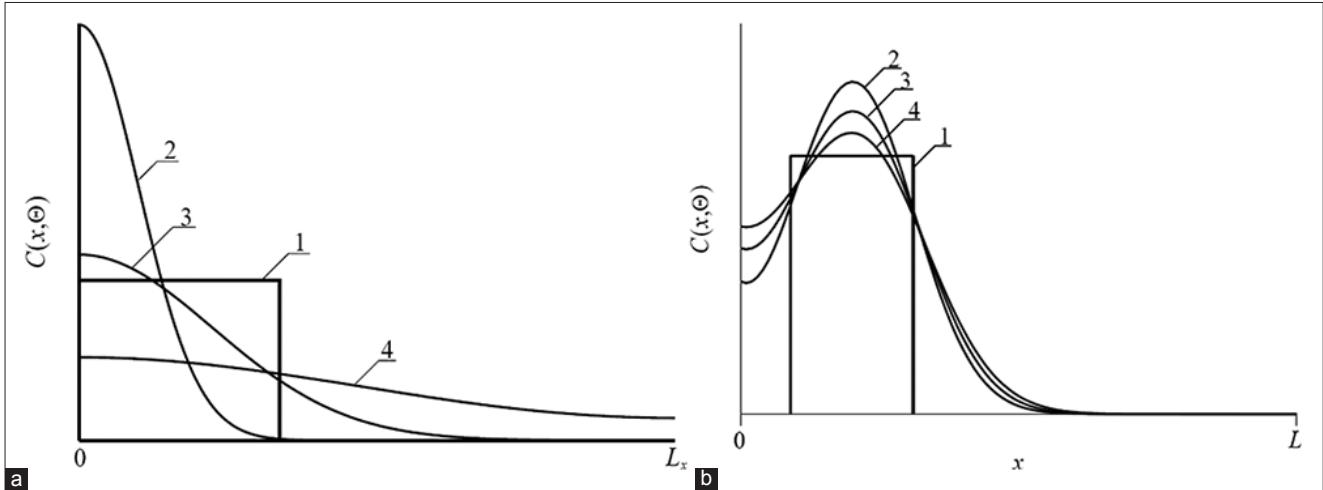


**Figure 3:** (a) Distributions of concentration of dopant, infused in average section of epitaxial layer of heterostructure from Figure 1 in direction parallel to interface between epitaxial layer and substrate of heterostructure. Difference between values of dopant diffusion coefficients increases with increasing of number of curves. Value of dopant diffusion coefficient in this section is smaller than the value of dopant diffusion coefficient in the nearest sections. (b) Calculated distributions of implanted dopant in epitaxial layers of heterostructure. Solid lines are spatial distributions of implanted dopant in system of two epitaxial layers. Dashed lines are spatial distributions of implanted dopant in one epitaxial layer. Annealing time increases with increasing of number of curves

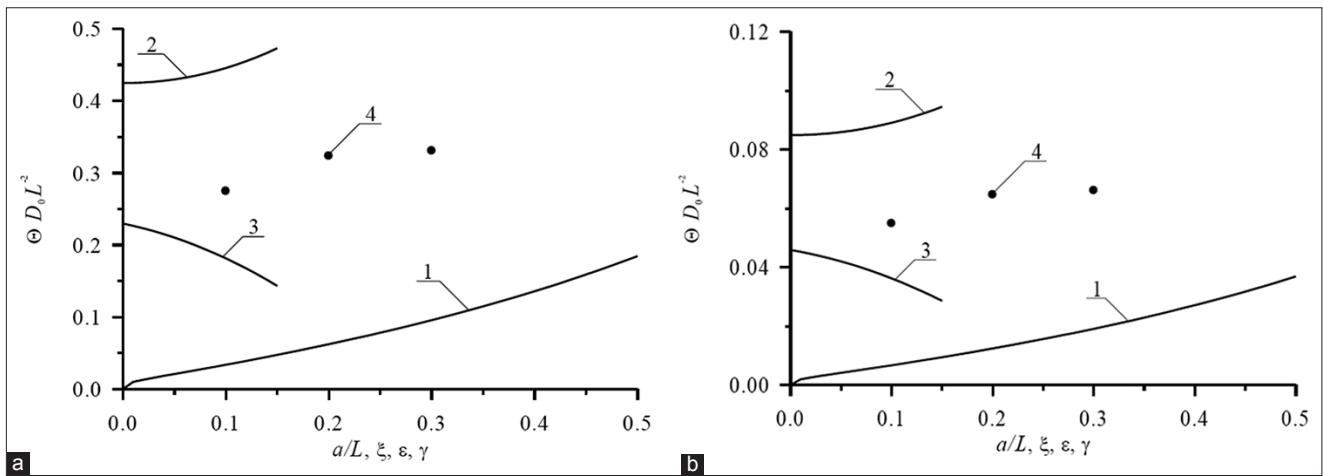
approach one should optimize annealing of dopant and/or radiation defects. To do the optimization, we used recently introduced criterion.<sup>[26-34]</sup> The optimization based on approximation real distribution by step-wise function  $\psi(x,y,z)$  [Figure 4]. Farther, the required values of optimal annealing time have been calculated by minimization the following mean squared error:

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left[ C(x, y, z, \Theta) - \psi(x, y, z) \right]^2 dz dy dx \quad (12)$$

We show optimal values of annealing time as functions of parameters on Figure 5. It is known that standard step of manufactured ion-doped structures is annealing of radiation defects. In the ideal case, after finishing the annealing dopant achieves interface between layers of heterostructure. If the dopant has no enough time to achieve the interface, it is practicable to anneal the dopant additionally. Figure 5b shows the described dependences of optimal values of additional annealing time for the same parameters as for Figure 5b. Necessity to anneal radiation defects leads to smaller values of optimal



**Figure 4:** (a) Distributions of concentration of infused dopant in depth of heterostructure from Figure 1 for different values of annealing time (curves 2–4) and idealized step-wise approximation (curve 1). Increasing of the number of curve corresponds to increasing of annealing time. (b) Distributions of concentration of implanted dopant in depth of heterostructure from Figure 1 for different values of annealing time (curves 2–4) and idealized step-wise approximation (curve 1). Increasing number of curve corresponds to increasing of annealing time



**Figure 5:** (a) Dimensionless optimal annealing time of infused dopant as a function of several parameters. Curve 1 describes the dependence of the annealing time on the relation  $a/L$  and  $\xi = \gamma = 0$  for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 describes the dependence of the annealing time on value of parameter  $\epsilon$  for  $a/L=1/2$  and  $\xi = \gamma = 0$ . Curve 3 describes the dependence of the annealing time on value of parameter  $\xi$  for  $a/L=1/2$  and  $\epsilon = \gamma = 0$ . Curve 4 describes the dependence of the annealing time on value of parameter  $\gamma$  for  $a/L=1/2$  and  $\epsilon = \xi = 0$ . (b) Dimensionless optimal annealing time of implanted dopant as a function of several parameters. Curve 1 describes the dependence of the annealing time on the relation  $a/L$  and  $\xi = \gamma = 0$  for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 describes the dependence of the annealing time on value of parameter  $\epsilon$  for  $a/L=1/2$  and  $\xi = \gamma = 0$ . Curve 3 describes the dependence of the annealing time on value of parameter  $\xi$  for  $a/L=1/2$  and  $\epsilon = \gamma = 0$ . Curve 4 describes the dependence of the annealing time on value of parameter  $\gamma$  for  $a/L = 1/2$  and  $\epsilon = \xi = 0$

annealing of implanted dopant in comparison with optimal annealing time of infused dopant.

## CONCLUSIONS

In this paper, we introduce an approach to increase integration rate of element of an operational amplifier circuit. The approach gives us possibility to decrease area of the elements with smaller increasing of the element's thickness.

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## APPENDIX

Equations for the functions  $\tilde{I}_{ijk}(\chi, \eta, \varphi, \vartheta)$  and  $\tilde{V}_{ijk}(\chi, \eta, \varphi, \vartheta)$ ,  $i \geq 0, j \geq 0, k \geq 0$  and conditions for them

$$\begin{aligned}
\frac{\partial \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] \\
\frac{\partial \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right]; \\
\frac{\partial \tilde{I}_{i00}(\chi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{i00}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{i00}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{i00}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] + \sqrt{\frac{D_{0I}}{D_{0V}}} \\
&\times \left\{ \frac{\partial}{\partial \chi} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{i-100}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right] + \frac{\partial}{\partial \eta} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{i-100}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right] \right. \\
&\left. + \frac{\partial}{\partial \varphi} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{i-100}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right] \right\}, \quad i \geq 1, \\
\frac{\partial \tilde{V}_{i00}(\chi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{i00}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{i00}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{i00}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] + \frac{\partial}{\partial \chi} \left[ g_V(\chi, \eta, \varphi, T) \right. \\
&\times \frac{\partial \tilde{V}_{i-100}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \left. \right] \sqrt{\frac{D_{0V}}{D_{0I}}} + \sqrt{\frac{D_{0V}}{D_{0I}}} \frac{\partial}{\partial \eta} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{i-100}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right] + \frac{\partial}{\partial \varphi} \left[ g_V(\chi, \eta, \varphi, T) \right. \\
&\times \frac{\partial \tilde{V}_{i-100}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \left. \right] \sqrt{\frac{D_{0V}}{D_{0I}}}, \quad i \geq 1, \\
\frac{\partial \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] \\
&- \left[ 1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T) \right] \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) \\
\frac{\partial \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] \\
&- \left[ 1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T) \right] \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta); \\
\frac{\partial \tilde{I}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] \\
&- \left[ 1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T) \right] \left[ \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) + \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta) \right] \\
\frac{\partial \tilde{V}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{020}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] \\
&- \left[ 1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T) \right] \left[ \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) + \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta) \right];
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] \\
&- [1 + \varepsilon_{I,I} g_{I,I}(\chi, \eta, \varphi, T)] \tilde{I}_{000}^2(\chi, \eta, \varphi, \vartheta) \\
\frac{\partial \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] \\
&- [1 + \varepsilon_{I,I} g_{I,I}(\chi, \eta, \varphi, T)] \tilde{V}_{000}^2(\chi, \eta, \varphi, \vartheta); \\
\frac{\partial \tilde{I}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] + \sqrt{\frac{D_{0I}}{D_{0V}}} \\
&\times \left\{ \frac{\partial}{\partial \chi} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right] + \frac{\partial}{\partial \eta} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right] + \frac{\partial}{\partial \varphi} \left[ g_I(\chi, \eta, \varphi, T) \right. \right. \\
&\times \left. \left. \frac{\partial \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right] \right\} - [\tilde{I}_{100}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) + \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{100}(\chi, \eta, \varphi, \vartheta)] \times \\
&\times [1 + \varepsilon_{I,I} g_{I,I}(\chi, \eta, \varphi, T)] \\
\frac{\partial \tilde{V}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{110}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] + \\
&+ \sqrt{\frac{D_{0V}}{D_{0I}}} \left\{ \frac{\partial}{\partial \chi} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right] + \frac{\partial}{\partial \eta} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right] + \right. \\
&+ \left. \frac{\partial}{\partial \varphi} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right] \right\} - [1 + \varepsilon_{V,V} g_{V,V}(\chi, \eta, \varphi, T)] \times \\
&\times [\tilde{V}_{100}(\chi, \eta, \varphi, \vartheta) \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) + \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{I}_{100}(\chi, \eta, \varphi, \vartheta)]; \\
\frac{\partial \tilde{I}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] - \\
&- [1 + \varepsilon_{I,I} g_{I,I}(\chi, \eta, \varphi, T)] \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta) \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \\
\frac{\partial \tilde{V}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{002}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] - \\
&- [1 + \varepsilon_{V,V} g_{V,V}(\chi, \eta, \varphi, E)] \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta); \\
\frac{\partial \tilde{I}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] +
\end{aligned}$$

$$\begin{aligned}
 & + \sqrt{\frac{D_{0I}}{D_{0V}}} \left\{ \frac{\partial}{\partial \chi} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right] + \frac{\partial}{\partial \eta} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right] + \right. \\
 & \left. + \frac{\partial}{\partial \varphi} \left[ g_I(\chi, \eta, \varphi, T) \frac{\partial \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right] \right\} - [1 + \varepsilon_I g_I(\chi, \eta, \varphi, T)] \tilde{I}_{100}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) \\
 \frac{\partial \tilde{V}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{101}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] + \\
 & + \sqrt{\frac{D_{0V}}{D_{0I}}} \left\{ \frac{\partial}{\partial \chi} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \right] + \frac{\partial}{\partial \eta} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \right] + \right. \\
 & \left. + \frac{\partial}{\partial \varphi} \left[ g_V(\chi, \eta, \varphi, T) \frac{\partial \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \right] \right\} - [1 + \varepsilon_V g_V(\chi, \eta, \varphi, T)] \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{100}(\chi, \eta, \varphi, \vartheta); \\
 \frac{\partial \tilde{I}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0I}}{D_{0V}}} \left[ \frac{\partial^2 \tilde{I}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{I}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{I}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] - \tilde{I}_{010}(\chi, \eta, \varphi, \vartheta) \times \\
 & \times [1 + \varepsilon_{I,I} g_{I,I}(\chi, \eta, \varphi, T)] \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) - [1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, T)] \tilde{I}_{001}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) \\
 \frac{\partial \tilde{V}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \vartheta} &= \sqrt{\frac{D_{0V}}{D_{0I}}} \left[ \frac{\partial^2 \tilde{V}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \chi^2} + \frac{\partial^2 \tilde{V}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \eta^2} + \frac{\partial^2 \tilde{V}_{011}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi^2} \right] - \tilde{V}_{010}(\chi, \eta, \varphi, \vartheta) \times \\
 & \times [1 + \varepsilon_{V,V} g_{V,V}(\chi, \eta, \varphi, T)] \tilde{V}_{000}(\chi, \eta, \varphi, \vartheta) - [1 + \varepsilon_{I,V} g_{I,V}(\chi, \eta, \varphi, t)] \tilde{I}_{000}(\chi, \eta, \varphi, \vartheta) \tilde{V}_{001}(\chi, \eta, \varphi, \vartheta); \\
 \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \Big|_{x=0} &= 0, \quad \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta)}{\partial \chi} \Big|_{x=1} = 0, \quad \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \Big|_{\eta=0} = 0, \quad \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta)}{\partial \eta} \Big|_{\eta=1} = 0, \\
 \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \Big|_{\varphi=0} &= 0, \quad \frac{\partial \tilde{\rho}_{ijk}(\chi, \eta, \varphi, \vartheta)}{\partial \varphi} \Big|_{\varphi=1} = 0 \quad (i \geq 0, j \geq 0, k \geq 0); \\
 \tilde{\rho}_{000}(\chi, \eta, \varphi, 0) &= f_\rho(\chi, \eta, \varphi) / \rho^*, \quad \tilde{\rho}_{ijk}(\chi, \eta, \varphi, 0) = 0 \quad (i \geq 1, j \geq 1, k \geq 1).
 \end{aligned}$$

Solutions of the above equations could be written as

$$\tilde{\rho}_{000}(\chi, \eta, \varphi, \vartheta) = \frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} F_{n\rho} c(\chi) c(\eta) c(\varphi) e_{n\rho}(\vartheta),$$

$$\text{Where, } F_{n\rho} = \frac{1}{\rho^*} \int_0^1 \cos(\pi n u) \int_0^1 \cos(\pi n v) \int_0^1 \cos(\pi n w) f_{n\rho}(u, v, w) dw dv du, \quad c_n(\chi) = \cos(\pi n \chi),$$

$$e_{nl}(\vartheta) = \exp(-\pi^2 n^2 \vartheta \sqrt{D_{0V}/D_{0I}}), \quad e_{nV}(\vartheta) = \exp(-\pi^2 n^2 \vartheta \sqrt{D_{0I}/D_{0V}});$$

$$\begin{aligned}
 \tilde{I}_{i00}(\chi, \eta, \varphi, \vartheta) &= -2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) c(\eta) c(\varphi) e_{nl}(\vartheta) \int_0^\vartheta e_{nl}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v) \int_0^1 \frac{\partial \tilde{I}_{i-100}(u, v, w, \tau)}{\partial u} \times \\
 & \times c_n(w) g_I(u, v, w, T) dw dv du d\tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) c(\eta) c(\varphi) e_{nl}(\vartheta) \int_0^\vartheta e_{nl}(-\tau) \int_0^1 c_n(u) \int_0^1 s_n(v) \times
 \end{aligned}$$

$$\begin{aligned}
 & \times \int_0^1 c_n(w) g_I(u, v, w, T) \frac{\partial \tilde{I}_{i-100}(u, v, w, \tau)}{\partial v} dw dv du d\tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) c(\eta) c(\varphi) e_{nI}(\vartheta) \int_0^\vartheta e_{nI}(-\tau) \\
 & \times \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 s_n(w) g_I(u, v, w, T) \frac{\partial \tilde{I}_{i-100}(u, v, w, \tau)}{\partial w} dw dv du d\tau, i \geq 1, \\
 \tilde{V}_{i00}(\chi, \eta, \varphi, \vartheta) & = -2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) c(\eta) c(\varphi) e_{nV}(\vartheta) \int_0^\vartheta e_{nI}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v) \int_0^1 g_V(u, v, w, T) \\
 & \times c_n(w) \frac{\partial \tilde{V}_{i-100}(u, \tau)}{\partial u} dw dv du d\tau - \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) c(\eta) c(\varphi) e_{nV}(\vartheta) \int_0^\vartheta e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 s_n(v) \\
 & \times 2\pi \int_0^1 c_n(w) g_V(u, v, w, T) \frac{\partial \tilde{V}_{i-100}(u, \tau)}{\partial v} dw dv du d\tau - 2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) c(\eta) c(\varphi) e_{nV}(\vartheta) \\
 & \times \int_0^\vartheta e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 s_n(w) g_V(u, v, w, T) \frac{\partial \tilde{V}_{i-100}(u, \tau)}{\partial w} dw dv du d\tau, i \geq 1,
 \end{aligned}$$

Where,  $s_n(\chi) = \sin(\pi n \chi)$ ;

$$\begin{aligned}
 \tilde{\rho}_{010}(\chi, \eta, \varphi, \vartheta) & = -2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{n\rho}(\vartheta) \int_0^\vartheta e_{n\rho}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) \\
 & \times [1 + \varepsilon_{I,V} g_{I,V}(u, v, w, T)] \tilde{I}_{000}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) dw dv du d\tau; \\
 \tilde{\rho}_{020}(\chi, \eta, \varphi, \vartheta) & = -2 \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{n\rho}(\vartheta) \int_0^\vartheta e_{n\rho}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) [1 + \varepsilon_{I,V} \\
 & \times g_{I,V}(u, v, w, T)] [\tilde{I}_{010}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) + \tilde{I}_{000}(u, v, w, \tau) \tilde{V}_{010}(u, v, w, \tau)] dw dv du d\tau; \\
 \tilde{\rho}_{001}(\chi, \eta, \varphi, \vartheta) & = -2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{n\rho}(\vartheta) \int_0^\vartheta e_{n\rho}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) \\
 & \times [1 + \varepsilon_{\rho,\rho} g_{\rho,\rho}(u, v, w, T)] \tilde{\rho}_{000}^2(u, v, w, \tau) dw dv du d\tau; \\
 \tilde{\rho}_{002}(\chi, \eta, \varphi, \vartheta) & = -2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{n\rho}(\vartheta) \int_0^\vartheta e_{n\rho}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) \\
 & \times [1 + \varepsilon_{\rho,\rho} g_{\rho,\rho}(u, v, w, T)] \tilde{\rho}_{001}(u, v, w, \tau) \tilde{\rho}_{000}(u, v, w, \tau) dw dv du d\tau; \\
 \tilde{I}_{110}(\chi, \eta, \varphi, \vartheta) & = -2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) c_n(\eta) c_n(\varphi) e_{nI}(\vartheta) \int_0^\vartheta e_{nI}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v) \int_0^1 c_n(u) \\
 & \times g_I(u, v, w, T) \frac{\partial \tilde{I}_{i-100}(u, v, w, \tau)}{\partial u} dw dv du d\tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) c_n(\eta) c_n(\varphi) e_{nI}(\vartheta)
 \end{aligned}$$

$$\begin{aligned}
 & \times \int_0^\vartheta e_{nl}(-\tau) \int_0^1 c_n(u) \int_0^1 s_n(v) \int_0^1 c_n(u) g_I(u, v, w, T) \frac{\partial \tilde{I}_{i-100}(u, v, w, \tau)}{\partial v} dwdvdu d\tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \\
 & \times \sum_{n=1}^{\infty} n e_{nl}(\vartheta) \int_0^\vartheta e_{nl}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 s_n(u) g_I(u, v, w, T) \frac{\partial \tilde{I}_{i-100}(u, v, w, \tau)}{\partial w} dwdvdu d\tau \\
 & \times c_n(\chi) c_n(\eta) c_n(\varphi) - 2 \sum_{n=1}^{\infty} c_n(\chi) e_{nl}(\vartheta) c_n(\eta) c_n(\varphi) \int_0^\vartheta e_{nl}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(v) [1 + \varepsilon_{I,V} \\
 & \times g_{I,V}(u, v, w, T)] [\tilde{I}_{100}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) + \tilde{I}_{000}(u, v, w, \tau) \tilde{V}_{100}(u, v, w, \tau)] dwdvdu d\tau \\
 & \tilde{V}_{110}(\chi, \eta, \varphi, \vartheta) = -2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) c_n(\eta) c_n(\varphi) e_{nV}(\vartheta) \int_0^\vartheta e_{nV}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v) \int_0^1 c_n(u) \times \\
 & \times g_V(u, v, w, T) \frac{\partial \tilde{V}_{i-100}(u, v, w, \tau)}{\partial u} dwdvdu d\tau - 2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) c_n(\eta) c_n(\varphi) e_{nV}(\vartheta) \times \\
 & \times \int_0^\vartheta e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 s_n(v) \int_0^1 c_n(u) g_V(u, v, w, T) \frac{\partial \tilde{V}_{i-100}(u, v, w, \tau)}{\partial v} dwdvdu d\tau - 2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \times \\
 & \times \sum_{n=1}^{\infty} n e_{nV}(\vartheta) \int_0^\vartheta e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 s_n(u) g_V(u, v, w, T) \frac{\partial \tilde{V}_{i-100}(u, v, w, \tau)}{\partial w} dwdvdu d\tau \times \\
 & \times c_n(\chi) c_n(\eta) c_n(\varphi) - 2 \sum_{n=1}^{\infty} c_n(\chi) e_{nl}(\vartheta) c_n(\eta) c_n(\varphi) \int_0^\vartheta e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(v) [1 + \varepsilon_{I,V} g_{I,V}(u, v, w, T)] \times \\
 & \times c_n(w) [\tilde{I}_{100}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) + \tilde{I}_{000}(u, v, w, \tau) \tilde{V}_{100}(u, v, w, \tau)] dwdvdu d\tau ; \\
 & \tilde{I}_{101}(\chi, \eta, \varphi, \vartheta) = -2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) c_n(\eta) c_n(\varphi) e_{nl}(\vartheta) \int_0^\vartheta e_{nl}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v) \int_0^1 g_I(u, v, w, T) \times \\
 & \times c_n(w) \frac{\partial \tilde{I}_{001}(u, v, w, \tau)}{\partial u} dwdvdu d\tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n c_n(\chi) c_n(\eta) c_n(\varphi) e_{nl}(\vartheta) \times \\
 & \times \int_0^1 s_n(v) \int_0^1 c_n(w) g_I(u, v, w, T) \frac{\partial \tilde{I}_{001}(u, v, w, \tau)}{\partial v} dwdvdu d\tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n e_{nl}(\vartheta) c_n(\chi) c_n(\eta) c_n(\varphi) \times \\
 & \times \int_0^\vartheta e_{nl}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 s_n(w) g_I(u, v, w, T) \frac{\partial \tilde{I}_{001}(u, v, w, \tau)}{\partial w} dwdvdu d\tau - 2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) \times \\
 & \times e_{nl}(\vartheta) \int_0^\vartheta e_{nl}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) [1 + \varepsilon_{I,V} g_{I,V}(u, v, w, T)] \\
 & \tilde{I}_{100}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) dwdvdu d\tau \\
 & \tilde{V}_{101}(\chi, \eta, \varphi, \vartheta) = -2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) c_n(\eta) c_n(\varphi) e_{nV}(\vartheta) \int_0^\vartheta e_{nV}(-\tau) \int_0^1 s_n(u) \int_0^1 c_n(v) \int_0^1 g_V(u, v, w, T) \times
 \end{aligned}$$

$$\begin{aligned}
 & \times c_n(w) \frac{\partial \tilde{V}_{001}(u, v, w, \tau)}{\partial u} dwdvdu d\tau - 2\pi \sqrt{\frac{D_{0V}}{D_{0I}}} \sum_{n=1}^{\infty} n c_n(\chi) c_n(\eta) c_n(\varphi) e_{nI}(\vartheta) \int_0^\vartheta e_{nV}(-\tau) \int_0^1 c_n(u) \times \\
 & \times \int_0^1 s_n(v) \int_0^1 c_n(w) g_I(u, v, w, T) \frac{\partial \tilde{I}_{001}(u, v, w, \tau)}{\partial v} dwdvdu d\tau - 2\pi \sqrt{\frac{D_{0I}}{D_{0V}}} \sum_{n=1}^{\infty} n e_{nI}(\vartheta) c_n(\chi) c_n(\eta) c_n(\varphi) \times \\
 & \times \int_0^\vartheta e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 s_n(w) g_V(u, v, w, T) \frac{\partial \tilde{V}_{001}(u, v, w, \tau)}{\partial w} dwdvdu d\tau - 2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) \times \\
 & \times e_{nV}(\vartheta) \int_0^\vartheta e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) [1 + \varepsilon_{I,V} g_{I,V}(u, v, w, T)] \tilde{I}_{100}(u, v, w, \tau) \\
 & \tilde{V}_{000}(u, v, w, \tau) dwdvdu d\tau; \\
 \tilde{I}_{011}(\chi, \eta, \varphi, \vartheta) & = -2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{nI}(\vartheta) \int_0^\vartheta e_{nI}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) \{ \tilde{I}_{000}(u, v, w, \tau) \times \\
 & \times [1 + \varepsilon_{I,I} g_{I,I}(u, v, w, T)] \tilde{I}_{010}(u, v, w, \tau) \\
 & + [1 + \varepsilon_{I,V} g_{I,V}(u, v, w, T)] \tilde{I}_{001}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) \} dwdvdu d\tau \\
 \tilde{V}_{011}(\chi, \eta, \varphi, \vartheta) & = -2 \sum_{n=1}^{\infty} c_n(\chi) c_n(\eta) c_n(\varphi) e_{nV}(\vartheta) \int_0^\vartheta e_{nV}(-\tau) \int_0^1 c_n(u) \int_0^1 c_n(v) \int_0^1 c_n(w) \{ \tilde{I}_{000}(u, v, w, \tau) \times \\
 & \times [1 + \varepsilon_{I,I} g_{I,I}(u, v, w, T)] \tilde{I}_{010}(u, v, w, \tau) \\
 & + [1 + \varepsilon_{I,V} g_{I,V}(u, v, w, T)] \tilde{I}_{001}(u, v, w, \tau) \tilde{V}_{000}(u, v, w, \tau) \} dwdvdu d\tau
 \end{aligned}$$

Equations for functions  $\Phi_{\rho_i}(x, y, z, t)$ ,  $i \geq 0$  to describe concentrations of simplest complexes of radiation defects.

$$\begin{aligned}
 \frac{\partial \Phi_{I0}(x, y, z, t)}{\partial t} & = D_{0\Phi I} \left[ \frac{\partial^2 \Phi_{I0}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 \Phi_{I0}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \Phi_{I0}(x, y, z, t)}{\partial z^2} \right] + \\
 & + k_{I,I}(x, y, z, T) I^2(x, y, z, t) - k_I(x, y, z, T) I(x, y, z, t) \\
 \frac{\partial \Phi_{V0}(x, y, z, t)}{\partial t} & = D_{0\Phi V} \left[ \frac{\partial^2 \Phi_{V0}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 \Phi_{V0}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \Phi_{V0}(x, y, z, t)}{\partial z^2} \right] + \\
 & + k_{V,V}(x, y, z, T) V^2(x, y, z, t) - k_V(x, y, z, T) V(x, y, z, t); \\
 \frac{\partial \Phi_{Ii}(x, y, z, t)}{\partial t} & = D_{0\Phi I} \left[ \frac{\partial^2 \Phi_{Ii}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 \Phi_{Ii}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \Phi_{Ii}(x, y, z, t)}{\partial z^2} \right] + \\
 & + D_{0\Phi I} \left\{ \frac{\partial}{\partial x} \left[ g_{\Phi I}(x, y, z, T) \frac{\partial \Phi_{Ii-1}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ g_{\Phi I}(x, y, z, T) \frac{\partial \Phi_{Ii-1}(x, y, z, t)}{\partial y} \right] + \right. \\
 & \left. + \frac{\partial}{\partial z} \left[ g_{\Phi I}(x, y, z, T) \frac{\partial \Phi_{Ii-1}(x, y, z, t)}{\partial z} \right] \right\}, \quad i \geq 1,
 \end{aligned}$$

$$\begin{aligned} \frac{\partial \Phi_{V_i}(x, y, z, t)}{\partial t} = & D_{0\Phi V} \left[ \frac{\partial^2 \Phi_{V_i}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 \Phi_{V_i}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 \Phi_{V_i}(x, y, z, t)}{\partial z^2} \right] + \\ & + D_{0\Phi V} \left\{ \frac{\partial}{\partial x} \left[ g_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{V_{i-1}}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ g_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{V_{i-1}}(x, y, z, t)}{\partial y} \right] + \right. \\ & \left. + \frac{\partial}{\partial z} \left[ g_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{V_{i-1}}(x, y, z, t)}{\partial z} \right] \right\}, i \geq 1; \end{aligned}$$

Boundary and initial conditions for the functions takes the form

$$\begin{aligned} \left. \frac{\partial \Phi_{\rho_i}(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial \Phi_{\rho_i}(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial \Phi_{\rho_i}(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad \left. \frac{\partial \Phi_{\rho_i}(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial \Phi_{\rho_i}(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial \Phi_{\rho_i}(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \quad i \geq 0; \quad \Phi_{\rho 0}(x, y, z, 0) = f_{\Phi \rho}(x, y, z), \end{aligned}$$

$$\Phi_{\rho i}(x, y, z, 0) = 0, \quad i \geq 1.$$

Solutions of the above equations could be written as

$$\begin{aligned} \Phi_{\rho 0}(x, y, z, t) = & \frac{1}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{n\Phi_{\rho}} c_n(x) c_n(y) c_n(z) e_{n\Phi_{\rho}}(t) + \frac{2}{L} \sum_{n=1}^{\infty} n c_n(x) c_n(y) c_n(z) \times \\ & \times e_{\Phi_{\rho^n}}(t) \int_0^t e_{\Phi_{\rho^n}}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) [k_I(u, v, w, T) I^2(u, v, w, \tau) - \\ & - k_I(u, v, w, T) I(u, v, w, \tau)] dwdvdu d\tau, \end{aligned}$$

$$\text{Where, } F_{n\Phi_{\rho}} = \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) f_{\Phi_{\rho}}(u, v, w) dw dv du,$$

$$e_{n\Phi_{\rho}}(t) = \exp \left[ -\pi^2 n^2 D_{0\Phi_{\rho}} t (L_x^{-2} + L_y^{-2} + L_z^{-2}) \right], \quad c_n(x) = \cos(\pi n x / L_x);$$

$$\begin{aligned} \Phi_{\rho i}(x, y, z, t) = & -\frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n c_n(x) c_n(y) c_n(z) e_{\Phi_{\rho^n}}(t) \int_0^t e_{\Phi_{\rho^n}}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} g_{\Phi_{\rho}}(u, v, w, T) \times \\ & \times c_n(w) \frac{\partial \Phi_{I_{\rho^{i-1}}}(u, v, w, \tau)}{\partial u} dw dv du d\tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n c_n(x) c_n(y) c_n(z) e_{\Phi_{\rho^n}}(t) \int_0^t e_{\Phi_{\rho^n}}(-\tau) \times \\ & \times \int_0^t e_{\Phi_{\rho^n}}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) g_{\Phi_{\rho}}(u, v, w, T) \frac{\partial \Phi_{I_{\rho^{i-1}}}(u, v, w, \tau)}{\partial v} dw dv du d\tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n \times \\ & \times e_{\Phi_{\rho^n}}(t) \int_0^t e_{\Phi_{\rho^n}}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) \frac{\partial \Phi_{I_{\rho^{i-1}}}(u, v, w, \tau)}{\partial w} g_{\Phi_{\rho}}(u, v, w, T) dw dv du d\tau \times \\ & \times c_n(x) c_n(y) c_n(z), \quad i \geq 1, \end{aligned}$$

where  $s_n(x) = \sin(\pi n x/L_x)$ .

Equations for the functions  $C_{ij}(x,y,z,t)$  ( $i \geq 0, j \geq 0$ ), boundary and initial conditions could be written as

$$\begin{aligned} \frac{\partial C_{00}(x,y,z,t)}{\partial t} &= D_{0L} \frac{\partial^2 C_{00}(x,y,z,t)}{\partial x^2} + D_{0L} \frac{\partial^2 C_{00}(x,y,z,t)}{\partial y^2} + D_{0L} \frac{\partial^2 C_{00}(x,y,z,t)}{\partial z^2}; \\ \frac{\partial C_{i0}(x,y,z,t)}{\partial t} &= D_{0L} \left[ \frac{\partial^2 C_{i0}(x,y,z,t)}{\partial x^2} + \frac{\partial^2 C_{i0}(x,y,z,t)}{\partial y^2} + \frac{\partial^2 C_{i0}(x,y,z,t)}{\partial z^2} \right] + \\ &+ D_{0L} \frac{\partial}{\partial x} \left[ g_L(x,y,z,T) \frac{\partial C_{i-10}(x,y,z,t)}{\partial x} \right] + D_{0L} \frac{\partial}{\partial y} \left[ g_L(x,y,z,T) \frac{\partial C_{i-10}(x,y,z,t)}{\partial y} \right] + \\ &+ D_{0L} \frac{\partial}{\partial z} \left[ g_L(x,y,z,T) \frac{\partial C_{i-10}(x,y,z,t)}{\partial z} \right], i \geq 1; \\ \frac{\partial C_{01}(x,y,z,t)}{\partial t} &= D_{0L} \frac{\partial^2 C_{01}(x,y,z,t)}{\partial x^2} + D_{0L} \frac{\partial^2 C_{01}(x,y,z,t)}{\partial y^2} + D_{0L} \frac{\partial^2 C_{01}(x,y,z,t)}{\partial z^2} + \\ &+ D_{0L} \frac{\partial}{\partial x} \left[ \frac{C_{00}^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)} \frac{\partial C_{00}(x,y,z,t)}{\partial x} \right] + D_{0L} \frac{\partial}{\partial y} \left[ \frac{C_{00}^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)} \frac{\partial C_{00}(x,y,z,t)}{\partial y} \right] + \\ &+ D_{0L} \frac{\partial}{\partial z} \left[ \frac{C_{00}^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)} \frac{\partial C_{00}(x,y,z,t)}{\partial z} \right]; \\ \frac{\partial C_{02}(x,y,z,t)}{\partial t} &= D_{0L} \frac{\partial^2 C_{02}(x,y,z,t)}{\partial x^2} + D_{0L} \frac{\partial^2 C_{02}(x,y,z,t)}{\partial y^2} + D_{0L} \frac{\partial^2 C_{02}(x,y,z,t)}{\partial z^2} + \\ &+ D_{0L} \left\{ \frac{\partial}{\partial x} \left[ C_{01}(x,y,z,t) \frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^\gamma(x,y,z,T)} \frac{\partial C_{00}(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ C_{01}(x,y,z,t) \frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^\gamma(x,y,z,T)} \frac{\partial C_{00}(x,y,z,t)}{\partial y} \right] \times \right. \\ &\times \left. \frac{\partial C_{00}(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ C_{01}(x,y,z,t) \frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^\gamma(x,y,z,T)} \frac{\partial C_{00}(x,y,z,t)}{\partial z} \right] \right\} + \\ &\times \frac{\partial C_{00}(x,y,z,t)}{\partial y} \left] + \frac{\partial}{\partial z} \left[ C_{01}(x,y,z,t) \frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^\gamma(x,y,z,T)} \frac{\partial C_{00}(x,y,z,t)}{\partial z} \right] \right\} + D_{0L} \left\{ \frac{\partial}{\partial x} \left[ \frac{C_{00}^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)} \times \right. \right. \\ &\times \left. \left. \frac{\partial C_{00}(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{C_{00}^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)} \frac{\partial C_{00}(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ \frac{C_{00}^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)} \frac{\partial C_{00}(x,y,z,t)}{\partial z} \right] \right\}; \\ \frac{\partial C_{11}(x,y,z,t)}{\partial t} &= D_{0L} \frac{\partial^2 C_{11}(x,y,z,t)}{\partial x^2} + D_{0L} \frac{\partial^2 C_{11}(x,y,z,t)}{\partial y^2} + D_{0L} \frac{\partial^2 C_{11}(x,y,z,t)}{\partial z^2} + \\ &+ \left\{ \frac{\partial}{\partial x} \left[ C_{10}(x,y,z,t) \frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^\gamma(x,y,z,T)} \frac{\partial C_{00}(x,y,z,t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ C_{10}(x,y,z,t) \frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^\gamma(x,y,z,T)} \times \right. \right. \\ &\times \left. \left. \frac{\partial C_{00}(x,y,z,t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ C_{10}(x,y,z,t) \frac{C_{00}^{\gamma-1}(x,y,z,t)}{P^\gamma(x,y,z,T)} \frac{\partial C_{00}(x,y,z,t)}{\partial z} \right] \right\} D_{0L} + \end{aligned}$$

$$\begin{aligned}
 & + D_{0L} \left\{ \frac{\partial}{\partial x} \left[ \frac{C_{00}^r(x, y, z, t)}{P^r(x, y, z, T)} \frac{\partial C_{10}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{C_{00}^r(x, y, z, t)}{P^r(x, y, z, T)} \frac{\partial C_{10}(x, y, z, t)}{\partial y} \right] + \right. \\
 & \left. + \frac{\partial}{\partial z} \left[ \frac{C_{00}^r(x, y, z, t)}{P^r(x, y, z, T)} \frac{\partial C_{10}(x, y, z, t)}{\partial z} \right] \right\} + D_{0L} \left\{ \frac{\partial}{\partial x} \left[ g_L(x, y, z, T) \frac{\partial C_{01}(x, y, z, t)}{\partial x} \right] + \right. \\
 & \left. + \frac{\partial}{\partial y} \left[ g_L(x, y, z, T) \frac{\partial C_{01}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ g_L(x, y, z, T) \frac{\partial C_{01}(x, y, z, t)}{\partial z} \right] \right\}; \\
 \frac{\partial C_{ij}(x, y, z, t)}{\partial x} \Big|_{x=0} & = 0, \quad \frac{\partial C_{ij}(x, y, z, t)}{\partial x} \Big|_{x=L_x} = 0, \quad \frac{\partial C_{ij}(x, y, z, t)}{\partial y} \Big|_{y=0} = 0, \quad \frac{\partial C_{ij}(x, y, z, t)}{\partial y} \Big|_{y=L_y} = 0, \\
 \frac{\partial C_{ij}(x, y, z, t)}{\partial z} \Big|_{z=0} & = 0, \quad \frac{\partial C_{ij}(x, y, z, t)}{\partial z} \Big|_{z=L_z} = 0, \quad i \geq 0, j \geq 0; \\
 C_{00}(x, y, z, 0) & = f_C(x, y, z), \quad C_{ij}(x, y, z, 0) = 0, \quad i \geq 1, j \geq 1.
 \end{aligned}$$

Functions  $C_{ij}(x, y, z, t)$  ( $i \geq 0, j \geq 0$ ) could be approximated by the following series during solutions of the above equations:

$$C_{00}(x, y, z, t) = \frac{F_{0C}}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t).$$

$$\text{Here, } e_{nC}(t) = \exp \left[ -\pi^2 n^2 D_{0C} t \left( \frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right) \right], \quad F_{nC} = \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} f_C(u, v, w) c_n(w) dw dv du;$$

$$\begin{aligned}
 C_{i0}(x, y, z, t) & = -\frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} g_L(u, v, w, T) \times \\
 & \times c_n(w) \frac{\partial C_{i-10}(u, v, w, \tau)}{\partial u} dw dv du d\tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \times \\
 & \times \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) g_L(u, v, w, T) \frac{\partial C_{i-10}(u, v, w, \tau)}{\partial v} dw dv du d\tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n F_{nC} e_{nC}(t) \times \\
 & \times c_n(x) c_n(y) c_n(z) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) g_L(u, v, w, T) \frac{\partial C_{i-10}(u, v, w, \tau)}{\partial w} dw dv du d\tau, \quad i \geq 1;
 \end{aligned}$$

$$\begin{aligned}
 C_{01}(x, y, z, t) & = -\frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \times \\
 & \times \frac{C_{00}^r(u, v, w, \tau)}{P^r(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} dw dv du d\tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \times \\
 & \times \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^r(u, v, w, \tau)}{P^r(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial v} dw dv du d\tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n e_{nC}(t) \times
 \end{aligned}$$

$$\begin{aligned}
 & \times F_{nC} c_n(x) c_n(y) c_n(z) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial w} dwdvdu d\tau; \\
 C_{02}(x, y, z, t) = & -\frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \times \\
 & \times C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} dwdvdu d\tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} F_{nC} c_n(x) c_n(y) \times \\
 & \times n c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial v} \times \\
 & \times c_n(w) dwdvdu d\tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \times \\
 & \times \int_0^{L_z} s_n(w) C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial w} dwdvdu d\tau - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n c_n(x) \times \\
 & \times F_{nC} c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) C_{01}(u, v, w, \tau) \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} \times \\
 & \times \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} dwdvdu d\tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \times \\
 & \times \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial v} dwdvdu d\tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n \times \\
 & \times F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) C_{01}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \times \\
 & \times \frac{\partial C_{00}(u, v, w, \tau)}{\partial w} dwdvdu d\tau - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \times \\
 & \times n \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{01}(u, v, w, \tau)}{\partial u} dwdvdu d\tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} c_n(x) e_{nC}(t) \times \\
 & \times F_{nC} c_n(y) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{01}(u, v, w, \tau)}{\partial v} dwdvdu d\tau \times \\
 & \times n c_n(z) - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) \times \\
 & \times \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{01}(u, v, w, \tau)}{\partial w} dwdvdu d\tau; \\
 C_{11}(x, y, z, t) = & -\frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) \times
 \end{aligned}$$

$$\begin{aligned}
& \times g_L(u, v, w, T) \frac{\partial C_{01}(u, v, w, \tau)}{\partial u} dwdvdu d\tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \times \\
& \times \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) g_L(u, v, w, T) \frac{\partial C_{01}(u, v, w, \tau)}{\partial v} dwdvdu d\tau - \frac{2\pi}{L_x L_y L_z^2} \times \\
& \times \sum_{n=1}^{\infty} n e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) g_L(u, v, w, T) \frac{\partial C_{01}(u, v, w, \tau)}{\partial w} dwdvdu d\tau \times \\
& \times F_{nC} c_n(x) c_n(y) c_n(z) - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \int_0^{L_y} c_n(v) \times \\
& \times n \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{10}(u, v, w, \tau)}{\partial u} dwdvdu d\tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) \times \\
& \times c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{10}(u, v, w, \tau)}{\partial v} dwdvdu d\tau - \\
& - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) \frac{C_{00}^\gamma(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \times \\
& \times \frac{\partial C_{10}(u, v, w, \tau)}{\partial w} dwdvdu d\tau - \frac{2\pi}{L_x^2 L_y L_z} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} s_n(u) \times \\
& \times \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) C_{10}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial u} dwdvdu d\tau - \frac{2\pi}{L_x L_y^2 L_z} \sum_{n=1}^{\infty} n \times \\
& \times F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \int_0^{L_y} s_n(v) \int_0^{L_z} c_n(w) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial v} \times \\
& \times C_{10}(u, v, w, \tau) dwdvdu d\tau - \frac{2\pi}{L_x L_y L_z^2} \sum_{n=1}^{\infty} n F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t) \int_0^t e_{nC}(-\tau) \int_0^{L_x} c_n(u) \times \\
& \times \int_0^{L_y} c_n(v) \int_0^{L_z} s_n(w) C_{10}(u, v, w, \tau) \frac{C_{00}^{\gamma-1}(u, v, w, \tau)}{P^\gamma(u, v, w, T)} \frac{\partial C_{00}(u, v, w, \tau)}{\partial w} dwdvdu d\tau .
\end{aligned}$$