ISSN 2581-3463

AJMS
Asian Journal of Mathematical Sciences

RESEARCH ARTICLE

On the Application of a Classical Fixed Point Method in the Optimization of a Multieffect Evaporator [2010 MSC: 46B25]

Eziokwu C. Emmanuel

Department of Mathematics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria

Received: 10-09-2018; Revised: 10-10-2018; Accepted: 10-01-2019

ABSTRACT

This work on classical optimization reveals the Newton's fixed point iterative method as involved in the computation of extrema of convex functions. Such functions must be differentiable in the Banach space such that their solution exists in the space on application of the Newton's optimization algorithm and convergence to the unique point is realized. These results analytically were carried as application into the optimization of a multieffect evaporator which reveals the feasibility of theoretical and practical optimization of the multieffect evaporator.

Key words: Non-linear Programming, convex subsets and the continuous function, Euler–Lagrange equation, newton's optimization algorithm, convergence, multieffect evaporator

INTRODUCTION

The non-linear programming problem for (P) is defined for (P) if $K=\{v \in X/(\varphi_i(v)) \leq 0,\ 1 \leq I \leq m',\ \varphi_i(v) = 0,\ m'+1 \leq 1 \leq m\}$. If φ_i and J are convex functionals, then (P) is called a convex programming while (P) is a quadratic programming if for $X = R^n$,

$$K = \left\{v \in R^n / \varphi_i(v) \leq d; 1 \leq i \leq m\right\}$$

$$J(v) = \frac{1}{2} Av, v - b, v,$$

Where, $A = \{a_{ij}\}$, an $n \times n$ positive definite matrix and $\varphi_i(v) = \sum_{j=1}^{n} a_{ij} v_{ji}$.

Definition [Peterson, J and Bayazitoglu, Y. (1991)]: Let A be a subset of a normed space X and f a real valued function on A. f is said to have a local or relative minimum (maximum) at $x_0 \in A$ if there is an open sphere $S_r(x_0)$ of X such that $f(x_0) \leq f(x)\left(f(x) \leq f(x_0)\right)$ holds for all $x \in S_r(x_0) \cap A$. If f has either a relative minimum or a relative maximum at x_0, then f is said to have a relative extremum.

Theorem 1.1 [Kaliventzeif, B (1991)]: Let $f : X \to R$ be a Gateaux differentiable functional at $x_0 \in X$ and f have a local extremum at x_0. Then, $Df(x_0)t = 0$ for all $t \in X$.

Proof

For every $t \in X$, the function $f(x_0 + \alpha t)$ (of a real variable function) has a local extremum at $\alpha = 0$. Since it is differentiable at 0, it follows from ordinary calculus that

$$\left[\frac{d}{d\alpha} f(x_0 + \alpha t)\right]_{\alpha=0} = 0$$

This means that $Df(x_0)t = 0$ for all $t \in X$ which proves the theorem.

Remark 1.1: Given a real-valued function on a solution of (P) on a convex set K and if f is a Gateaux differentiable at x_0, then

$$Df(x_0)(x - x_0) \geq 0;\ \forall x \in K$$

Address for correspondence:
Eziokwu C. Emmanuel,
E-mail: okereemm@yahoo.com

Theorem 1.2 (Existence of Solution in *R*) [Kaliventzeif, B. (1991)]: Let K be a non-empty closed convex subset of R^n and $J: R^n \to R$ a continuous function which is coercive if K is unbounded. Then, there exists at least one solution of (P).

Proof

Let $\{U_k\}$ be a minimizing sequence of J; that is a sequence satisfying conditions $u_k \in K$ for every integer k and $\lim_{k\to\infty} \underset{u\in K}{Inf}\, J(u)$.

This sequence is necessarily bounded since the functional J is coercive so that it is possible to find a subsequence $\{U_{k'}\}$ which converges to an element $v \in K$ (K being closed). Since J is continuous, $J(u) = \lim_{k'\to\infty} J(U_{k'}) = \underset{v\in K}{Inf}\, J(v)$, which proves the existence of a solution of (P).

Theorem 1.3 (Existence of Solution in Infinite Dimensional Hilbert Space):[43] Let K be a non-empty convex closed subset of a separable Hilbert space H and $J: H \to R$ a convex, continuous functional which is coercive if K is unbounded. Then, (*P*) has at least one solution. Proof[8] [see A.H. Siddiqi (1993)].

MINIMIZATION OF ENERGY FUNCTIONAL

In this section, we employ the use of classical calculus of variation which is a special case of (P) where we look for the extremum of functional of the type

$$J(u) = \int_a^b F(x,u,u')\,dx, \quad \left(u'(x) = \frac{du}{dx}\right) \tag{2.1}$$

Which is twice differentiable on $[a,b]$ and F has a continuous partial derivative with respect to u and u'. Also considered is the functional

$$J(v) = \frac{1}{2}\alpha(v,v) - F(v) \tag{2.2}$$

Where $a(.,.)$ is a bilinear and continuous form on a Hilbert space X and F is an element of the dual space X^* of X which is an energy functional on a quadratic functional.

Theorem 2.1:[29] A necessary condition for the functional $J(u)$ to have an extremum at u is that u must satisfy the Euler–Lagrange equation

$$\frac{\partial F}{\partial u} - \frac{d}{dx}\left(\frac{\partial F}{\partial u'}\right) = 0$$

In $[a,b]$ with the boundary condition in $u(a) = \alpha$ and $u(b) = \beta$.

Proof

Let $u(a) = 0$ and $u(b) = 0$, then

$$J(u+\alpha v) - J(u) = \int_a^b \begin{bmatrix} F(x,u+\alpha v,u'+\alpha v) \\ -F(\alpha,u,u') \end{bmatrix} dx$$

Using the Taylor series expansion

$$\begin{aligned} &F(x,u+\alpha v,u'+\alpha v') = \\ &F(x,u,u') + \alpha\left(v\frac{\partial F}{\partial u} + v\frac{\partial F}{\partial u'}\right) \\ &+\frac{\alpha^2}{2!}\left(v\frac{\partial F}{\partial u} + v\frac{\partial F}{\partial u'}\right)^2 + \cdots \end{aligned} \tag{2.3}$$

It follows from (2.3) that

$$\begin{aligned} J(u+\alpha v) &= J(u) + \alpha dJ(u)(v) \\ &+\frac{\alpha^2}{2!} d^2 J(u)(v) + \ldots \end{aligned} \tag{2.4}$$

Where, the first and second Frechet differentials are given by

$$dJ(u)v = \int_a^b \left(u\frac{\partial F}{\partial u} + u'\frac{\partial F}{\partial u'}\right) dx$$

$$d^2 J(u)v = \int_a^b \left(v\frac{\partial F}{\partial u} + v'\frac{\partial F}{\partial u'}\right)^2 dx$$

The necessary condition for the functional J to have an extremum at u is that $JJ(u)v = 0$ for all $v \in C^2[a,b]$ such that $v(a) = v(b) = 0$ that is

$$0 = dJ(u)v = \int_a^b \left(v\frac{\partial F}{\partial u} + v'\frac{\partial F}{\partial u'}\right) dx \tag{2.5}$$

Integrating the second term in the integrand (2.5) by parts, we get

$$\int_a^b \left[\frac{\partial F}{\partial u}-\frac{d}{dx}\left(\frac{\partial F}{\partial u^{'}}\right)\right]vdx+\left[v\frac{\partial F}{\partial u^{'}}\right]_a^b=0$$

Since $v(a)=v(b)=0$, the boundary terms vanish and the necessary conditions become

$$\int_a^b \left[\frac{\partial F}{\partial u}-\frac{d}{dx}\left(\frac{\partial F}{\partial u^{'}}\right)\right]vd=0 \; for\,all\, v\in C^2[a,b]$$

For all functions $v\in C^2[a,b]$ vanishing at *a* and *b*. This is possible only if

$$\frac{\partial F}{\partial u}-\frac{d}{dx}\left(\frac{\partial F}{\partial u^{'}}\right)=0$$

Thus, the desired result is achieved.

Theorem 2.2: Let $\alpha(.,.)$ be coercive and symmetric, and K a non-empty closed convex subset of X. Then, (P) for J in (2.2) has a unique solution in K.

Proof

The bilinear form induces an inner product over the Hilbert space equivalent to the norm induced by the inner product of X. In fact, the equations imply that

$$\sqrt{\alpha}\,\|v\|\le(\alpha(v,v))^{\frac{1}{2}}\le\sqrt{\|\alpha\|}\,\|v\|$$

Since F is a linear continuous form with this new norm, the Riez representation theorem exists and has a unique element $u\in X$ such that $F(v)=\alpha(u,v)$ for every $u\in X$.

Hence,

$$J(v)=\frac{1}{2}\alpha(v,v)-\alpha(u,v)$$

$$=\frac{1}{2}\alpha(v-u,v-u)-\frac{1}{2}\alpha(u,u)$$

$=\frac{1}{2}v-u,v-u-\frac{1}{2}<u,u>$ for all $v\in K$ and a unique u.

Therefore, $\mathrm{Inf}_{v\in K}\,J(v)$ is equivalent to $Inf_{v\in K}\|v-u\|$. Thus, in the present situation, (P) amounts to looking for the projection x of the element u onto subset K. Therefore, (P) has a unique solution.

Optimization algorithm and the convergence theorems[1]

The iterative method for this research is the Newton's method stated below. For the function $F:U\subset R\to R,U$ the open subset of R, the Newton's method is

$$u_{k+1}=u_k-\frac{F(u_k)}{F^{'}(u_k)},\; k\ge 0,$$

u_0 an arbitrary starting point in the open set U. Hence, the sequence is defined by $u_{k+1}=u_k-\left\{F^{'}(u_k)\right\}^{-1}F(u_k)$

under the assumption that all the points lie in U and if $X=R^n, Y=R^n, F(u)=0$ is equivalent to

$F_1(u)=0,\quad u=(u_1,u_2,\ldots,u_n)\in R^n$

$F_2(u)=0$

$F_3(u)=0$

$F_n(u)=0$

Where, $F:R^n\to R,\; i=1,2,\ldots,n$

Theorem 2.2.1 (convergence): [2-7]Let X be a Banach Space, U an open subset of X,Y normed linear space, and $F:U\subset X\to Y$ differentiable over U. Suppose that there exists three constants α,β,γ such that $\alpha>0$ and

$$S_\alpha(u_0)=\left\{u\in X/u-u_0\le\alpha\right\}\subseteq U$$

i. $Sup_{k\ge0}\,Sup_{u\in S_\alpha(u_0)}\|A^{-1}{}_k\|_{B[X,Y]}\le\beta,$

$A_k(u)=A_k\in B[X,Y]$ is bijective

ii. $Sup_{k\ge0}Sup_{u^{'}\in S_\alpha(u_0)}\|F^{'}(x)-A_k(x)\|_{B[X,Y]}$

$\le\frac{\gamma}{\beta}and\,\gamma<1$

iii. $\| F(u_0) \| \leq \frac{\alpha}{\beta}(1-\gamma)$

Then, the sequence defined by

$$u_{k+1} = u_k - A^{-1}\left(u_{k'}\right)F\left(u_k\right), \qquad k \geq k' \geq 0$$

is entirely contained within the ball and converges to a zero of F in $S_x\left(u_0\right)$ which is unique. Furthermore,

$$\| u_k - u \| \leq \frac{\| u_1 - u_0 \|}{1-\gamma}\gamma^k$$

Theorem 2.2.2 (convergence): [9-15]Let X be a Banach space, U an open subset of $X.F : U \subset X \to Y$ and Y a normed space. Furthermore, let F be continuously differentiable over U. Suppose that u is a point of U such that

$F\left(u\right) = 0, A = F'\left(u\right) : X \to Y$ bounded linear and bijective

$$Sup_{k\geq 0} \| A_k - A \|_{B[X,Y]} \leq \frac{\lambda}{\| A^{-1} \|_{B[X,Y]}} \text{ and } \lambda < 1$$

Then, there exists a closed ball $S_r\left(u_0\right)$ with center u and radius r such that for every point $u_0 \in S_r\left(u\right)$, the sequence $\left\{U_k\right\}$ defined by

$$u_{k+1} = u_k - A^{-1}{}_k F\left(u_k\right), \quad k \geq 0$$

is constrained in $S_r\left(u\right)$ and converges to point u, which is the only zero of F in the ball $S_r\left(u\right)$. Furthermore, there exists a number γ such that

$$\gamma < 1 \text{ and } \| u_k - u \| \leq \gamma^k \| u_0 - u \|, \; k \geq 0$$

APPLICATION TO THE OPTIMIZATION OF A MULTIEFFECT EVAPORATOR

When a process requires an evaporation step, the problem of evaporator design needs serious examination. Although the subject of evaporation and the equipment to carry out evaporation have been studied and analyzed for many years, each application has to receive individual attention. No evaporation configuration and its equipment can be picked from a stock list and be expected to produce trouble-free operation.[16-19]

An engineer working on the selection of optimal evaporation equipment must list what is "known," "unknown," and "to be determined." Such analysis should at least include the following:

Known

- Production rate and analysis of product
- Feed flow rate, feed analysis, and feed temperature
- Available utilities (steam, water, gas, etc.)
- Disposition of condensate (location) and its purity
- Probable materials of construction.

Unknown

- Pressures, temperatures, solids, compositions, capacities, and concentrations
- Number of evaporator effects
- Amount of vapor leaving the last effect
- Heat transfer surface.

Features to be determined

- Best type of evaporator body and heater arrangement
- Filtering characteristics of any solid or crystals
- Equipment dimensions arrangement
- Separator elements for purity overhead vapors
- Materials, fabrication details, and instrumentation.

Utility consumption

- Steam
- Electric power
- Water
- Air.

In multiple effect evaporation, as shown in Figure 1a, the total capacity of the system of evaporation is no greater than that of a single effect evaporator having a heating surface equal to one effect and operating under the same terminal conditions. The amount of water vaporized per unit surface area in n effects is roughly $\frac{1}{n}$ that of a single effect. Furthermore, the boiling point elevation causes a loss of available temperature drop in every effect, thus reducing capacity. Why then are multiple effects often economic? It is

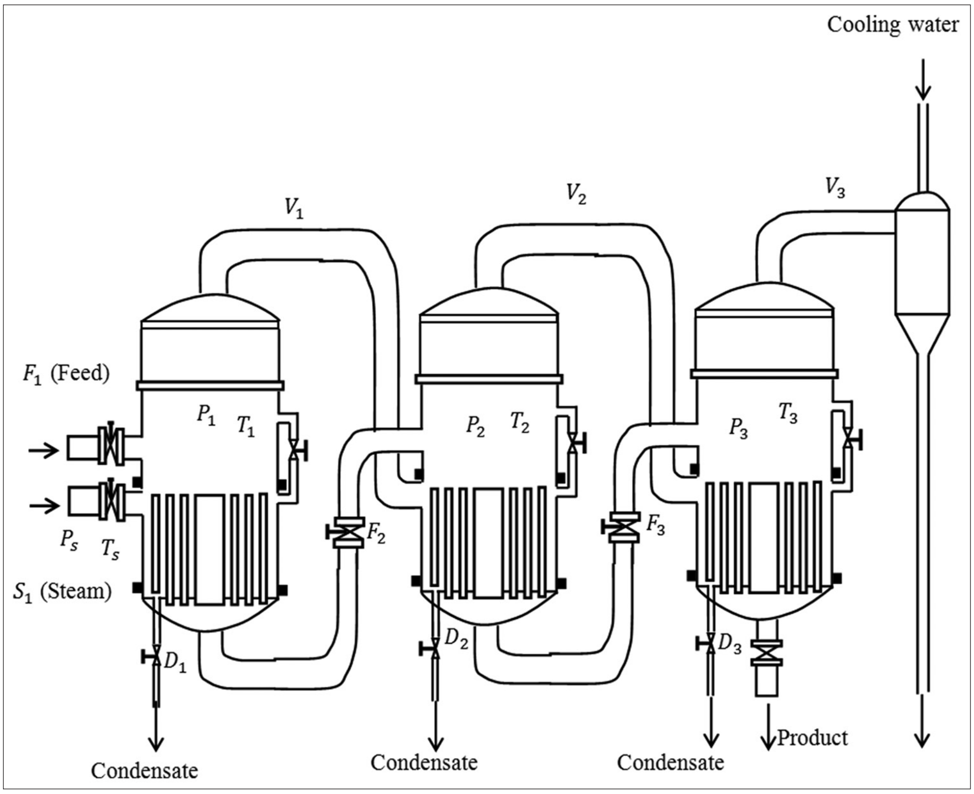

Figure 1a: Multiple effect evaporator with forward feed [Singiresu, S.R. (1996)]

because the cost of an evaporator per square foot of surface area decreases with total area (and asymptotically becomes a constant value) so that to achieve a given production, the cost of heat exchange can be balanced with the steam costs.[20-24] Steady-state mathematical models of single and multiple effect evaporators involving material energy balances can be found in McCabe *et al.* (1993), Yanniotis and Pilavachi (1996), and Esplugas and Mata (1983). The classical simplified optimization problem for evaporators (Schweyer, 1995) is to determine the most suitable number of effects given[25-32]

(1) An analytical expression for the fixed costs in terms of the number of effects n
(2) The steam (variable) cost also in terms of n.

Analytic differentiation yields an analytical solution for the optimal n^*, as shown here.

Assume we are concentrating an organic salt in the range of 0.1 to 1.0 wt% using a capacity of 0.1–10 million gallons/day. Initially, we treat the number of stages n as a continuous variable. Figure1b shows a single effect in the process. Before discussions of the capital and operating costs, we need to define the temperature driving force for heat transfer in Figure 1c.[33,34] By definition the log mean temperature difference ΔT_{lm} is

$$\Delta T_{lm} = \frac{T_i - T_d}{\text{In}\left(T_i / T_d\right)} \tag{a}$$

Let T_i be equal to constant K for a constant performance ratio P. Because $T_d = T_i - \frac{\Delta T_f}{n}$

$$\Delta T_{lm} = \frac{\frac{\Delta T_f}{n}}{\text{In}\left[\frac{K}{K} - \left(\frac{T_f}{n}\right)\right]} \tag{b}$$

Let $A =$ Condenser heat transfer areas ft^2

c_p = Liquid heat capacity, $1.05\frac{Btu}{\left(lb_m\right)\left(°F\right)}$

C_c = Cost per unit area of condenser, $\frac{6.25}{ft^2}$

C_E = Cost per evaporator (including partitions), $\frac{7000}{stage}$

C_s = cost of steam, $\$lb$ at the brine heater (first stages)

F_{out} = liquid flow out of evaporator, $\frac{lb}{h}$

$K = T_i$, a constant ($T_i = \Delta T - T_d$ at inlet)

n = number of stages

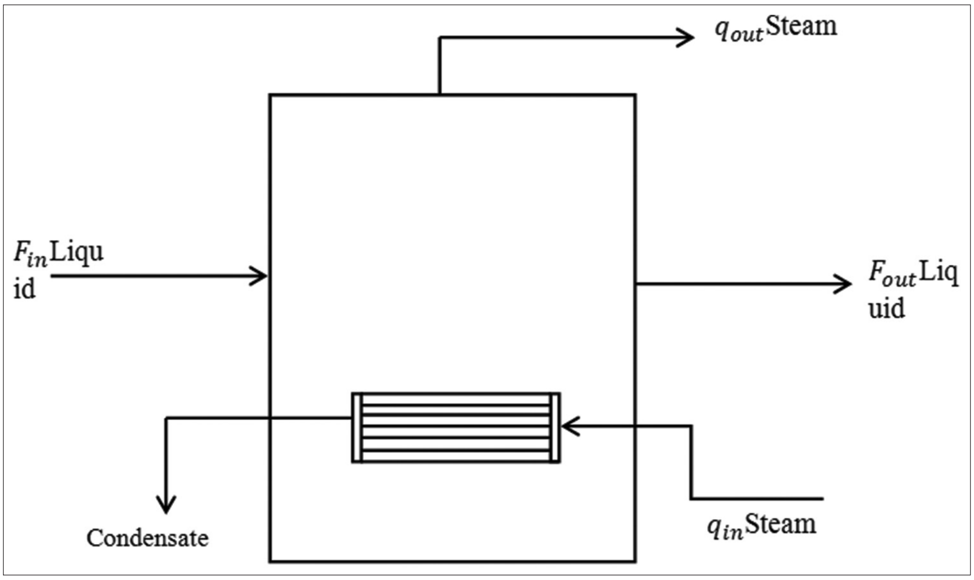

Figure 1b: Individual effect evaporator with forward feed

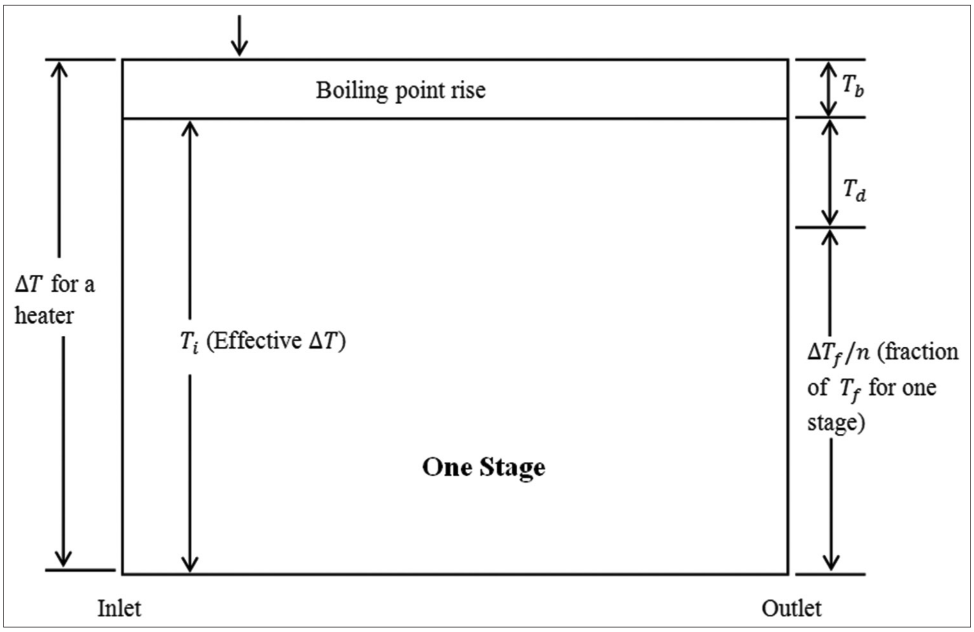

Figure 1c: Boiling Point effect

P = Performance ratio, lb of H_2O evaporated/Btu supplied to brine heater

Q = heat duty, $9.5\times10^8\,\frac{Btu}{h}$ (a constant)

q_e = total $lb\,H_2O$ evaporated/h

q_r = total lb steam used/h

r = Capital recovery factor

S = lb steam supplied/h

T_b = boiling point rise, $4.3°F$

ΔT_f = flash down range, $250°F$

U = overall heat transfer coefficient (assumed to be constant), $\frac{625Btu}{(ft^2)(h)(°F)}$

ΔH_{vap} = heat of vaporization of water, about $\frac{1000Btu}{lb}$

The optimum number of stages is n^*. For a constant performance ratio, the total cost of the evaporator is

$$f_1 = C_E n + C_c A \qquad (c)$$

For A, we introduce

$$A = \frac{Q}{U(\Delta T_{lm})}$$

Then, we differentiate f_1 in equation (c) with respect to n and set the resulting expression equal to zero (Q and U are constant)

$$C_E + C_c \frac{Q}{U}\left[\frac{\partial(1/\Delta T_{lm})}{\partial n}\right]_P = 0 \quad \text{(d)}$$

With the use of equation (b)

$$\left[\frac{\partial\left(1/\Delta T_{lm}\right)}{\partial n}\right]_P = \frac{1}{nK\left(1-\dfrac{\Delta T_f}{nK}\right)} - \frac{In\left(1-\Delta T_f\right)}{\Delta T_f} \quad \text{(e)}$$

Substituting equation (e) into (d) plus introducing the values of $Q, U, \Delta T_f, C_{E,}$ and C_c, we get

$$7000 - \left[\frac{(6.25)\left(9.5\times10^8\right)}{625}\right]$$

$$\left[\frac{1}{nK\left(1-\Delta T_f / nk\right)} + \frac{In\left(1-\Delta T_f / nK\right)}{\Delta T_f}\right] = 0$$

Rearranging

$$\frac{(625)(7000)(250)}{(6.25)\left(9.5\times10^8\right)} = 0.184 =$$

$$\frac{250}{nK-250} + \text{In}\left(1+\frac{250}{nK}\right) \quad \text{(f)}$$

In practice, as the evaporation plant size changes (for constant Q), the ratio of the stage condenser area cost to the unit evaporator cost remains essentially constant so that the number 0.184 is treated as a constant for all practical purposes. Equation (f) can be solved for nK for constant P.[35-40]

$$nK = 590 \quad \text{(g)}$$

Next, we eliminate K from equation (g) by replacing K with a function of P so that n becomes a function of P. The performance ratio (with constant liquid heat capacity at $347°\text{F}$) is defined as

$$P = \frac{\left(\Delta H_{vap}\right)\left(q_e\right)}{\left(F_{out} C_{pF} \Delta T_{heater}\right)_{\text{first stage}}} = \frac{1000}{1.05\left(4.3+K\right)} \frac{q_e}{F_{out}} \quad \text{(h)}$$

The ratio $\frac{q_e}{F}$ can be calculated from

$$\frac{q_e}{F_{out}} = 1 - \left(\frac{1194-322}{1194-70}\right)^{1.49} = 0.31$$

Where,

$$\Delta H_{vap}\left(355°\text{F}, 143\text{psi}\right) = 1194 Btu/lb$$

$$\Delta H_{liq} H_2 0\left(350°F\right) = 322 Btu/lb$$

$$\Delta H_{liq} H_2 0\left(100°F\right) = \frac{70Btu}{lb}$$

Equations (g) and (h) can be solved together to eliminate K and obtain the desired relation

$$\frac{300}{P} - 4.3 = \frac{590}{n^*} \quad \text{(i)}$$

Equation (i) shows how the boiling point rise ($T_b = 4.3°\text{F}$) and the number of stages affects the performance ratio.

Optimal performance ratio

The optimal plant operation can be determined by minimizing the total cost function, including steam cost, with respect to P (liquid pumping costs are negligible)

$$f_2 = \left[C_c A + C_E n\right] r + C_s S \quad \text{(j)}$$

$$rC_c = \frac{\partial A}{\partial P} + rC_E \frac{\partial n}{\partial P} + C_c \frac{\partial S}{\partial P} = 0 \quad \text{(k)}$$

The quantity for $\frac{\partial A}{\partial P}$ can be calculated using the equations already developed and can be expressed in terms of a ratio of polynomials in P such as

$$\frac{a\left(1+1/P\right)}{\left(1-bP\right)^2}$$

Where, a and b are determined by fitting experimental data. The relation for $\frac{\partial n}{\partial P}$ can be determined from equation (i). The relation for $\frac{\partial S}{\partial P}$ can be obtained from equation (l)

$$P = \frac{q_e}{Q} = \frac{q_e}{\Delta H_{vap}} = \frac{q_e}{1000S}$$

or

$$S\left(\frac{lb}{h}\right) = \frac{q_e}{1000P}$$

or

$$S(lb) = \frac{\alpha(8760)q_e}{1000P} \tag{l}$$

Where, α is the fraction of hours per year (8760) during which the system operates. Equation (k) given the cost cannot be explicitly solved for P^*, but P^* can be obtained by any effective root finding technique.

If a more complex mathematical model is employed to represent the evaporation process, you must shift from analytic to numerical methods. The material and enthalpy balances become complicated functions of temperature (and pressure). Usually, all of the system parameters are specified except for the heat transfer areas in each effect (n unknown variables) and the vapor temperatures in each effect excluding the last one ($n-1$ unknown variables). The model introduces n independent equations that serve as constraints, many of which are non-linear, plus non-linear relations among the temperatures, concentrations, and physical properties such as enthalpy and the heat transfer coefficient.[41-44]

The number of evaporators represents an integer-valued variable because many engineers use tables and graphs as well as equations for evaporator calculations.

REFERENCES

1. Siddiqi AH. Applied Functional Analysis. New York: Marcel Dekker Inc.; 2004.
2. Athier G, Floquet P, Pibouleau L, Domenech S. Process optimization by simulation annealing and NLP procedures. Application to heat exchanger network synthesis. Comput Chem Eng 1997;21 Suppl: S475-80.
3. Briones V, Kokossis A. A new approach for the optimal retrofit of heat exchanger Networks. Comput Chem Eng 1996;20 Suppl: S43-8.
4. Chidume C. Abdulsalam centre for theoretic physics. In: Geometric Properties of Banach Spaces and Nonlinear Iterations. Trieste Italy, London: Springe Verlag Limited; 2009.
5. Chaudhuri PD, Diweker UM, Logsdon JS. An automated approach for the optimal design of heat exchangers. Ind Eng Chem Res 1999;36:3685-93.
6. Cichelli MT, Brinn MS. How to design optimum heat exchanger. Chem Eng 1956;4:196.
7. Ciric AR, Floudas CA. Heat exchanger network synthesis without documentation. Comput Chem Eng 1999;15:385-96.
8. Colmenares TR, Seider WD. Heat and power integration of chemical processes. AIChE J 1987;33:898-915.
9. Cornellisen RL, Hiss GG. Thermodynamic optimization of a heat exchanger. Int J Heat Mass Transfer 1999;42:951-9.
10. Daichendt MM, Grossmann IE. Prelimnary screening procedure for MINLP synthesis of process systems II. Heat exchanger networks. Comput Chem Eng 1986;18:679-710.
11. Anderson ED. Linear Optimization Theory Methods and Extensions. Department of Management. Denmark: Odense University; 1998.
12. Esplugas S, Mata J. Calculator design of multistage evaporators. Chem Eng 1983;59:120-14.
13. Fabbri G. Heat transfer optimization in internally finned tubes under laminar flow conditions. Int J Heat Mass Transfer 1998;41:1243-53.
14. Riesz F, Sz-Nagy B. Functional Analysis. New York: Dover Publications Inc.; 1990.
15. Georgiadis MC, Papergeorgiou LG, Macchietto S. Optimal cleaning policies in heat exchanger networks under rapid fouling. Ind Chem Res 2000;39:441-54.
16. Gunderson T, Naess L. The synthesis of cost optimal heat exchanger networks. An industrial review of the state of the art. Comput Chem Eng 1988;12:503-30.
17. Taha HA. Operations Research, an Introduction. New Delhi, India: Pearson Education; 2002.
18. Ikegami Y, Bejan A. On the thermodynamic optimization of power plants with heat transfer and fluid flow irreversibilities. J Solar Energy Eng 1998;120:139-44.
19. Jegede FO, Polley GT. Capital cost targets for networks with non-uniform heat transfer specifications. Comput Chem Eng 1992;16:477.
20. Kaliventzeff B. Mixed integer nonlinear programming and its application to the management of utility networks. Eng Optim 1991;18:183-207.
21. Lang YD, Biegler LT, Grossmann IE. Simultaneous optimization and heat integration with process simulators. Comput Chem Eng 1988;12:311-28.
22. Linnhoff B. Pinch analysis a state of the art overview. Chem Eng Res Des 1993;71:503-23.
23. Luus R. Optimization of heat exchanger networks. Ind Eng Chem Res 1993;32:2633-5.
24. McAdams WH. Heat Transmission. New York: McGraw-Hill; 1942.
25. McCabe WL, Smith J, Harriott P. Unit Operations in Chemical Engineering. 5th ed. New York: McGraw-Hill; 1993.
26. Altman M. Contractors and Contractor Directions, Theory Applications. New York: Mercel Dekker Inc.; 1977.
27. Peters M, Timmerhaus K. Plant Design and Economics for Chemical Engineers. 4th ed. New York: McGraw-Hill; 1991.
28. Peterson J, Bayazitoglu Y. Optimization of cost subject to uncertainty constraints in experimental fluid flow and heat transfer. J Heat Transfer 1991;113:314-20.
29. Quesada I, Grossmann IE. Global optimization algorithm for heat exchanger networks. Ind Eng Chem Res 1993;32:487-99.
30. Sundaram RK. A First Course in Optimization Theory. Cambridge: Cambridge University Press; 1971.
31. Reppich M, Zagermann S. A new design method for

segementally baffled heat exchangers. Comput Chem Eng 1995;19 Suppl: S137-42.
32. Sama DA. Economic Optimum LMTD at Heat Exchangers. Houston, Texas: AIChe National Meeting; 1983.
33. Schweyer HE. Process Engineering Economics. New York: McGraw-Hill; 1995. p. 214.
34. Rao SS. Engineering Optimization Theory and Practice. New Delhi: New Age International Publishers, A.H. Pint Park; 1996.
35. Steinmeyer DE. Process Energy Conservation Kirk Othmer Encyclopedia. 3rd ed. New York: Wiley; 1984.
36. Swearingen JS, Ferguson JE. Optimized power recovery from waste heat. Chem Eng Prog 1983;79:66-70.
37. Tarrer AR, Lim HC, Koppel LB. Finding the economically optimum heat exchanger. Chem Eng 1971;79:79-94.
38. Tayal MC, Fu Y, Diwekar UM. Optimal design of heat exchangers: A genetic algorithm framework. Ind Eng Chem Res 1999;38:456-67.
39. Hogar TP, Kimmelblam DM. Optimization of Chemical Process. New York: McGraw-Hill Higher Education; 2001.
40. Yanniotis S, Pilavachi PA. Mathematical modeling and experimental validation of an absorber-driven multiple effect evaporator. Chem Eng Technol 1996;19:448-55.
41. Yee TF, Grossmann IE, Kravanja Z. Simultaneous optimization models for heat integration-I. Area and energy targeting and modeling of multistream exchangers. Comput Chem Eng 1990;14:1165-83.
42. Yeh R. Errors in one dimensional fin optimization problem for convective heat transfer. Int J Heat Mass Transfer 1996;39:3075-8.
43. Zamora JM, Grossmann IE. A global MINLP optimization algorithm for the synthesis of heat exchanger networks with no stream splits. Comput Chem Eng 1998;22:367-84.
44. Zhu XX, Asante ND. Diagnosis and optimization approach for heat exchanger network retrofit. AIChE J 1999;45:1488-503.