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RESEARCH ARTICLE

New Method for Finding an Optimal Solution of Generalized Fuzzy Transportation Problems

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ABSTRACT

In this paper, a proposed method, namely, zero average method is used for solving fuzzy transportation problems by assuming that a decision-maker is uncertain about the precise values of the transportation costs, demand, and supply of the product. In the proposed method, transportation costs, demand, and supply are represented by generalized trapezoidal fuzzy numbers. To illustrate the proposed method, a numerical example is solved. The proposed method is easy to understand and apply to real-life transportation problems for the decision-makers.

Key words: Fuzzy transportation problem, generalized trapezoidal fuzzy number, ranking function, zero average method

INTRODUCTION

The transportation problem is an important network structured in linear programming (LP) problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in the problem is to find the least total transportation cost of a commodity to satisfy demands at destinations using available supplies at origins. Transportation problem can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, employment scheduling, and many others. In general, transportation problems are solved with the assumptions that the transportation costs and values of supplies and demands are specified in a precise way, that is, in a crisp environment. However, in many cases, the decision-makers have no crisp information about the coefficients belonging to the transportation problem. In these cases, the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy sets, and the fuzzy transportation problem (FTP) appears in a natural way.

The basic transportation problem was originally developed by Hitchcock.[1] The transportation problems can be modeled as a standard LP problem, which can then be solved by the simplex method. However, due to its very special mathematical structure, it was recognized early that the simplex method applied to the transportation problem can be made quite efficient in terms of how to evaluate the necessary simplex method information (variable to enter the basis, variable to leave the basis and optimality conditions). Charnes and Cooper[2] developed a stepping stone method which provides an alternative way of determining the simplex method information. Dantzig and Thapa[3] used the simplex method to the transportation problem as the primal simplex transportation method. An Initial Basic Feasible Solution for the transportation problem can be obtained by using the Northwest corner rule, row minima, column minima, matrix minima, or Vogel's approximation method. The Modified Distribution Method is useful for finding the optimal solution to the transportation problem. In general, the transportation problems are solved with the assumptions that the coefficients or cost

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parameters are specified in a precise way, that is, in a crisp environment.

In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence, etc. Sometimes, it is not possible to get relevant precise data for the cost parameter. This type of imprecise data is not always well represented by a random variable selected from a probability distribution. Fuzzy number[4] may represent the data. Hence, the fuzzy decision-making method is used here.

Zimmermann[5] showed that solutions obtained by the fuzzy LP method and are always efficient. Subsequently, Zimmermann's fuzzy LP has developed into several fuzzy optimization methods for solving transportation problems. Chanas *et al*.[6] presented a fuzzy LP model for solving transportation problems with a crisp cost coefficient, fuzzy supply, and demand values. Chanas and Kuchta[7] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers and developed an algorithm for obtaining the optimal solution. Saad and Abbas[8] discussed the solution algorithm for solving the transportation problem in fuzzy environment. Liu and Kao[9] described a method for solving FTP based on extension principle. Gani and Razak[10] presented a two stage cost minimizing FTP, in which supplies and demands are trapezoidal fuzzy numbers. A parametric approach is used to obtain a fuzzy solution, and the aim is to minimize the sum of the transportation costs in two stages. Lin[11] introduced a genetic algorithm to solve the transportation problem with fuzzy objective functions. Dinagar and Palanivel[12] investigated FTP, with the aid of trapezoidal fuzzy numbers. Fuzzy modified that the distribution method is proposed to find the optimal solution in terms of fuzzy numbers. Pandian and Natarajan[13] proposed a new algorithm, namely, fuzzy zero point method for finding a fuzzy optimal solution for a FTP, where the transportation cost, supply, and demand are represented by trapezoidal fuzzy numbers. Edward Samuel[14-16] showed the unbalanced FTPs without converting into a balanced one getting an optimal solution, where the transportation cost, demand, and supply are represented by a triangular fuzzy number. Edward Samuel[17] proposed algorithmic approach to unbalanced FTPs, where the transportation cost, demand, and supply are represented by a triangular fuzzy number. Edward Samuel[16] proposed a new procedure for solving generalized trapezoidal FTP, where precise values of the transportation costs only, but there is no uncertainty about the demand and supply. Edward Samuel[18] proposed a dual based approach for solving unbalanced FTP, where precise values of the transportation cost only, but there is no uncertainty about the demand and supply. In the literature, there are many other problems which are solved by various methods via generalized trapezoidal fuzzy numbers, for example, see.[19,20-25]

In this paper, a proposed method, namely, zero average method (ZAM), is used for solving a special type of FTP by assuming that a decision-maker is uncertain about the precise values of transportation costs, demand, and supply of the product. In the proposed method, ZAM transportation costs, demand, and supply of the product are represented by generalized trapezoidal fuzzy numbers. To illustrate the proposed method ZAM, a numerical example is solved. The proposed method ZAM is easy to understand and to apply in real-life transportation problems for the decision-makers.

PRELIMINARIES

In this section, some basic definitions, arithmetic operations, and an existing method for comparing generalized fuzzy numbers are presented.

Definition 1:[11]A fuzzy set Ã defined on the universal set of real numbers $\Re$ is said to be fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}}(x): \Re \to [0,1]$ is continuous.

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.

(iii) $\mu_{\tilde{A}}(x)$ Strictly increasing on [a, b] and strictly decreasing on [c, d]

(iv) $\mu_{\tilde{A}}(x)$=1 for all $x \in [b,c], where\ a < b < c < d.$

Definition 2: [11]A fuzzy number Ã = (a, b, c, d) is said to be trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)}, & a \le x < b, \\ 1 \quad , & b \le x \le c, \\ \dfrac{(x-d)}{(c-d)}, & c < x \le d, \\ o \quad , & otherwise. \end{cases}$$

Definition 3: [12]A fuzzy set Ã defined on the universal set of real numbers $\Re$, is said to be a generalized fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}}(x)$: $\Re \rightarrow [0, \omega]$ is continuous.

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.

(iii) $\mu_{\tilde{A}}(x)$ Strictly increasing on [a, b] and strictly decreasing on [c, d]

(iv) $\mu_{\tilde{A}}(x) = \omega$, for all $x \in [b, c], where \ 0 < \omega \leq 1$.

Definition 4: [12]A fuzzy number Ã = (a, b, c, d; ω) is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \omega \dfrac{(x-a)}{(b-a)}, & a \leq x < b, \\ \omega \quad , & b \leq x \leq c, \\ \omega \dfrac{(x-d)}{(c-d)}, & c < x \leq d, \\ o \quad , & otherwise. \end{cases}$$

Arithmetic operations

In this section, arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on the universal set of real numbers $\Re$, are presented.[12,18] Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; \omega_2)$ are two generalized trapezoidal fuzzy numbers; then, the following is obtained.

i. $\tilde{A}_1 + \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(\omega_1, \omega_2))$,

ii. $\tilde{A}_1 - \tilde{A}_2 = (a_1 - d_2, b_1 - b_2, c_1 - c_2, d_1 - a_2; \min(\omega_1, \omega_2))$,

iii. $\tilde{A}_1 \otimes \tilde{A}_2 \cong (a, b, c, d; \min(\omega_1, \omega_2))$

Where

$a = \min(a_1a_2, a_1d_2, a_2d_1, d_1d_2)$, $b = \min(b_1b_2, b_1c_2, c_2b_2, c_1c_2)$,

$c = \max(b_1b_2, b_1c_2, c_1b_2, c_1c_2)$, $d = \max(a_1a_2, a_1d_2, a_2d_1, d_1d_2)$

iv. $$\lambda \tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; \omega_1), & \lambda > 0, \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; \omega_1), & \lambda < 0. \end{cases}$$

Ranking function

An efficient approach for comparing the fuzzy numbers is by the use of the ranking function,[18,9,13] $\Re : F(\Re) \rightarrow \Re$, where F($\Re$) is a set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into the real line, where a natural order exists, that is,

(i) $\tilde{A} >_{\Re} \tilde{B}$ if and only if $\Re(\tilde{A}) > \Re(\tilde{B})$

(ii) $\tilde{A} <_{\Re} \tilde{B}$ if and only if $\Re(\tilde{A}) < \Re(\tilde{B})$

(iii) $\tilde{A} =_{\Re} \tilde{B}$ if and only if $\Re(\tilde{A}) = \Re(\tilde{B})$

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; \omega_2)$ be two generalized trapezoidal fuzzy numbers and $\omega = \min(\omega_1, \omega_2)$.

$$Then \ \Re(\tilde{A}_1) = \frac{\omega(a_1 + b_1 + c_1 + d_1)}{4} \ and$$

$$\Re(\tilde{A}_2) = \frac{\omega(a_2 + b_2 + c_2 + d_2)}{4}.$$

PROPOSED METHOD

In this section, we present the proposed method, namely ZAM for finding a fuzzy optimal solution, in which the transportation costs, demand, and supply are represented by the generalized trapezoidal fuzzy numbers.

Step 1. Check whether the given FTP is balanced, go to step 3 if-else go to step 2

Step 2.

- If the given FTP is unbalanced, then any one of the following two cases may arise
- If the total demand exceeds total supply, then go to case1 if-else go to case 2.

Case 1.

- Locate the smallest fuzzy cost in each row of the given cost table and then subtract that from each fuzzy cost of that row
- Convert into balanced one and then replace the dummy fuzzy cost of the largest unit fuzzy transportation cost in the reduced matrix obtain from case 1 of a
- In the reduced matrix obtained from case 1 of b, locate the smallest fuzzy cost in each column, and then subtract that from each fuzzy cost of that column and then go to step 4.

Case 2.

- Locate the smallest fuzzy cost in each column of the given cost table and then subtract that from each fuzzy cost of that column
- Convert into balanced one and then replace the dummy fuzzy cost of the largest unit fuzzy transportation cost in the reduced matrix obtain from case 2 of a
- In the reduced matrix obtained from case 2 of b, locate the smallest fuzzy cost in each row, and then subtract that from each fuzzy cost of that row and then go to step 4.

Step 3.

- Locate the smallest fuzzy cost in each row of the given cost table and then subtract that from each fuzzy cost of that row, and
- In the reduced matrix obtained from 3(a), locate the smallest fuzzy cost in each column and then subtract that from each fuzzy cost of that column.

Step 4. Select the smallest fuzzy costs (not equal to zero) in the reduced matrix obtained from step 2 or step 3 and then subtract it by selected smallest fuzzy cost only. Each row and column now have at least one fuzzy zero value; if there is no fuzzy zero, then repeat step 3.

Step 5.

- Select the fuzzy zero (row-wise) and count the number of fuzzy zeros (excluding selected one) in row and column and record as a subscript of selected zero. Repeat the process for all zeros in the matrix
- Now, choose the value of subscript is minimum and allocate maximum to that cell. If a tie occurs for some fuzzy zero values, then find the average of demand and supply value and then choose with minimum one.

Step 6. Adjust the supply and demand and cross out the satisfied the row or column.

Step 7. Check whether the resultant matrix possesses at least one fuzzy zero in each and column. If not, repeat step 3, otherwise go to step 8.

Step 8. Repeat step 3 and step 5 to step 7 until all the demand and supply are exhausted.

Step 9. Compute total fuzzy transportation cost for the feasible allocation from the original fuzzy cost table.

Remark 1: If there is a tie in the values of the average of demand and supply, then calculate their corresponding row and column value and select one with minimum.

NUMERICAL EXAMPLE

To illustrate, the proposed method, namely, ZAM, the following FTP is solved

Table 1: Fuzzy Transportation Problem for Example 1

Source	D_1	D_2	D_3	Supply (a_i)
S_1	(11, 13, 14, 18; 0.5)	(20, 21, 24, 27; 0.7)	(14, 15, 16, 17; 0.4)	13
S_2	(6, 7, 8, 11; 0.2)	(9, 11, 12, 13; 0.2)	(20, 21, 24, 27; 0.7)	20
S_3	(14, 15, 17, 18; 0.4)	(15, 16, 18, 19; 0.5)	(10, 11, 12, 13; 0.6)	5
Demand (b_j)	12	15	11	

Example 1: Table 1 gives the availability of the product available at three sources and their demand at three destinations, and the approximate unit transportation cost of the product from each source to each destination is represented by a generalized trapezoidal fuzzy number. Determine the fuzzy optimal transportation of the products such that the total transportation cost is minimum.

Using step 1, $\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j = 38$, so the chosen problem is a balanced FTP.

Iteration 1. Using step 3 and step 4, we get:

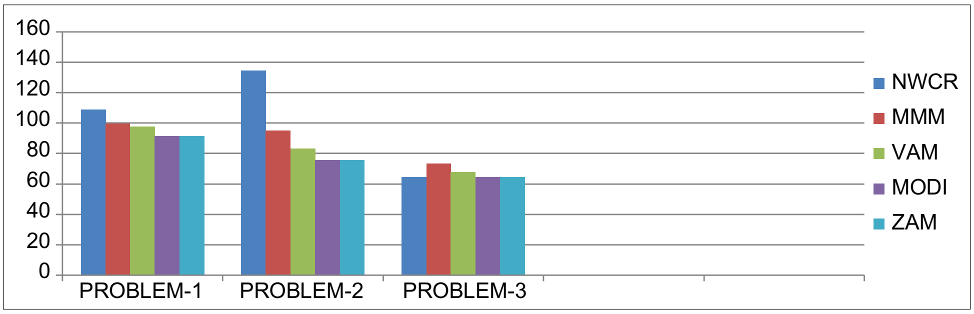
Source	D_1	D_2	D_3	Supply (a_i)
S_1	(−14, −2, 2, 14; 0.5)	(−5, 2, 8, 18; 0.2)	(−16, −4, 4, 16; 0.4)	13
S_2	(−12, −2, 2, 12; 0.2)	(−9, −2, 2, 9; 0.2)	(6, 12, 18, 24; 0.2)	20
S_3	(−6, 2, 7, 15; 0.4)	(−5, −1, 4, 11; 0.2)	(−6, −2, 2, 6; 0.6)	5
Demand (b_j)	12	15	11	

Iteration 2. Using step 5 and step 6, we get:

Source	D_1	D_2	D_4	Supply (a_i)
S_1	(−14, −2, 2, 14; 0.5)	(−5, 2, 8, 18; 0.2)	(−16, −4, 4, 16; 0.4)	13
S_2	(−12, −2, 2, 12; 0.2)	(−9, −2, 2, 9; 0.2)	(6, 12, 18, 24; 0.2)	20
S_3	*	*	(−6, −2, 2, 6; 0.6)5	*
Demand (b_j)	12	15	6	

Iteration 3. Using step 7 and step 8, allocate the values:

Source	D_1	D_2	D_4	Supply (a_i)
S_1	(11, 13, 14, 18; 0.5)^{7}	(20, 21, 24, 27; 0.7)	(14, 15, 16, 17; 0.4)^{6}	*
S_2	(6, 7, 8, 11; 0.2)^{5}	(9, 11, 12, 13; 0.2)^{15}	(20, 21, 24, 27; 0.7)	*
S_3	(14, 15, 17, 18; 0.4)	(15, 16, 18, 19; 0.5)	(10, 11, 12, 13; 0.6)^{5}	*
Demand (b_j)	*	*	*	



Graph 1: Graphical comparisons of ZAM with various existing methods

Iteration 4. Using step 9 we get:

Source	D_1	D_2	D_3	Supply (a_i)
S_1	(11, 13, 14, 18; 0.5)^{7}	(20, 21, 24, 27; 0.7)	(14, 15, 16, 17; 0.4)^{6}	*
S_2	(6, 7, 8, 11; 0.2)^{5}	(9, 11, 12, 13; 0.2)^{15}	(20, 21, 24, 27; 0.7)	*
S_3	(14, 15, 17, 18; 0.4)	(15, 16, 18, 19; 0.5)	(10, 11, 12, 13; 0.6)^{5}	*
Demand (b_j)	*	*	*	

The minimum fuzzy transportation cost is equal to 7 (11, 13, 14, 18; 0.5) + 6 (14, 15, 16, 17; 0.4) + 5 (6, 7, 8, 11; 0.2) +15 (9, 11, 12, 13; 0.2) + 5 (10, 11, 12, 13; 0.6) = (376, 436, 474, 543; 0.2). Therefore, the ranking function R(A) = 91.45

Results with normalization process

If all the values of the parameters used in problem 1 are first normalized and then the problem is solved by using the ZAM, then the fuzzy optimal value is is $\tilde{x}_0 = (376, 436, 474, 543; 1)$.

Results without normalization process

If all the values of the parameters of the same problem 1 are not normalized and then the problem is solved using the ZAM, then the fuzzy optimal value is $\tilde{x}_0 = (376, 436, 474, 543; .2)$.

Remark 2: Results with normalization process represent the overall level of satisfaction of decision-maker about the statement that minimum transportation cost will lie between 436 and 474 units as 100% while without normalization process, the overall level of satisfaction of the decision-maker for the same range is 20%. Hence, it is better to use generalized fuzzy numbers instead of normal fuzzy numbers obtained using the normalization process.

Table 2: Comparisons of ZAM with existing methods.

Row	Column	NWCR	MMM	VAM	MODI	ZAM
3	3	108.80	99.50	97.50	91.45	91.45
3	4	134.18	95.00	83.00	75.60	75.60
3	3	64.35	73.10	67.60	64.35	64.35

ZAM: Zero Average Method

Example 2:

Source	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	(11, 12, 13, 15; 0.5)	(16, 17, 19, 21; 0.6)	(28, 30, 34, 35; 0.7)	(4, 5, 8, 9; 0.2)	8
S_2	(49, 53, 55, 60; 0.8)	(18, 20, 21, 23; 0.4)	(18, 22, 25, 27; 0.6)	(25, 30, 35, 42; 0.7)	10
S_3	(28, 30, 34, 35; 0.7)	(2, 4, 6, 8; 0.2)	(36, 42, 48, 52; 0.8)	(6, 7, 9, 11; 0.3)	11
Demand (b_j)	4	7	6	12	

COMPARATIVE STUDY AND RESULT ANALYSIS

From the investigations and the results are given in Table 2, it clear that ZAM is better than **NWCR,**[6] **MMM,**[6] and **VAM**[26] for solving FTP and also, the solution of the FTP is given by ZAM that is an optimal solution.

Table 2 represents the solution obtained by NWCR,[6] MMM,[6] VAM,[26] MODI,[3] and ZAM. This data speaks the better performance of the proposed method. The graphical representation of the solution obtained by various methods of this performance, displayed in Graph 1.

CONCLUSION

In this paper, a new method, namely, ZAM, is proposed for finding an optimal solution of FTPs, in which the transportation costs, demand, and supply of the product are represented as generalized trapezoidal fuzzy numbers. The advantage of the proposed method is discussed, and a numerical example is solved to illustrate the ZAM. The ZAM is easy to understand and apply for solving the FTPs occurring in real-life situations.

REFERENCES

1. Kaur A, Kumar A. A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers. Appl Soft Comput 2012;12:1201-13.
2. Chanas S, Kolodziejckzy W, Machaj AA. A fuzzy approach to the transportation problem. Fuzzy Sets Syst 1984;13:211-21.
3. Chanas S, Kuchta D. A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. Fuzzy Sets Syst 1996;82:299-305.
4. Charnes A, Cooper WW. The stepping-stone method for explaining linear programming calculation in transportation problem. Manage Sci 1954;l:49-69.
5. Charnes A, Cooper WW, Henderson A. An Introduction to Linear Programming. New York: Willey; 1953. p. 113.
6. Chen SJ, Chen SM. Fuzzy risk analysis on the ranking of generalized trapezoidal fuzzy numbers. Appl Intell 2007;26:1-11.
7. Chen SM, Chen JH. Fuzzy risk analysis based on the ranking generalized fuzzy numbers withdifferent heights and different spreads. Expert Syst Appl 2009;36:6833-42.
8. Dantzig GB, Thapa MN. Linear Programming, Theory and Extensions. New Jersey: Princeton University Press, Springer; 1963.
9. Dinagar DS, Palanivel K. The transportation problem in fuzzy environment. Int J Algorithms Comput Math 2009;2:65-71.
10. Samuel AE, Venkatachalapathy M. A new dual based approach for the unbalanced fuzzy transportation problem. Appl Math Sci 2012;6:4443-55.
11. Samuel AE, Venkatachalapathy M. A new procedure for solving generalized trapezoidal fuzzy transportation problem. Adv Fuzzy Sets Syst 2012;12:111-25.
12. Samuel AE, Venkatachalapathy M. Improved zero point method for solving fuzzy transportation problems using ranking function. Far East J Math Sci 2013;75:85-100.
13. Gani A, Razak KA. Two stage fuzzy transportation problem. J Phys Sci 2006;10:63-9.
14. Hitchcock FL. The distribution of a product from several sources to numerouslocalities. J Math Phys 1941;20:224-30.
15. Kaufmann A, Gupta MM. Introduction to Fuzzy Arithmetics: Theory and Applications. New York: Van Nostrand Reinhold; 1991.
16. Lin FT. Solving the Transportation Problem with Fuzzy Coefficients Using Genetic Algorithms. India: Proceedings IEEE International Conference on Fuzzy Systems; 2009. p. 20-4.
17. Liou TS, Wang MJ. Ranking fuzzy number with integral values. Fuzzy Sets Syst 1992;50:247-55.
18. Liu ST, Kao C. Solving fuzzy transportation problems based on extension principle. Eur J Oper Res 2004; 153:661-74.
19. Mahapatra GS, Roy TK. Fuzzy multi-objective mathematical programming on reliability optimization model. Appl Math Comput 2006;174:643-59.
20. Pandian P, Natarajan G. A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems. Appl Math Sci 2010;4:79-90.
21. Reinfeld NV, Vogel WR. Mathematical Programming. Englewood Clifts, New Jersey: Prentice-Hall; 1958. p. 59-70.
22. Saad OM, Abbas SA. A parametric study on transportation problem under fuzzy environment. J Fuzzy Math 2003;11:115-24.
23. Zadeh LA. Fuzzy sets. Inform Control 1965;8:338-53.
24. Zimmermann HJ. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets Syst 1978;1:45-55.
25. Kumar SS, Raja P, Shanmugasundram P, Thota S. A new method to solving generalized fuzzy transportation problem-harmonic mean method. Int J Chem Math Phys 2020;4:51-6.
26. Samuel AE, Raja P, Thota S. A new technique for solving unbalanced intuitionistic fuzzy transportation problems. Appl Math Inf Sci 2020;14:459-65.