ISSN 2581-3463

AJMS
Asian Journal of Mathematical Sciences

RESEARCH ARTICLE

Investigation of Parameter Behaviors in Stationarity of Autoregressive and Moving Average Models through Simulations

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Received: 25-10-2020; Revised: 28-11-2020; Accepted: 15-12-2020

ABSTRACT

The most important assumption about time series and econometrics data is stationarity. Therefore, this study focuses on behaviors of some parameters in stationarity of autoregressive (AR) and moving average (MA) models. Simulation studies were conducted using R statistical software to investigate the parameter values at different orders (p) of AR and (q) of MA models, and different sample sizes. The stationary status of the p and q are, respectively, determined, parameters such as mean, variance, autocorrelation function (ACF), and partial autocorrelation function (PACF) were determined. The study concluded that the absolute values of ACF and PACF of AR and MA models increase as the parameter values increase but decrease with increase of their orders which as a result, tends to zero at higher lag orders. This is clearly observed in large sample size (n = 300). However, their values decline as sample size increases when compared by orders across the sample sizes. Furthermore, it was observed that the means values of the AR and MA models of first order increased with increased in parameter but decreased when sample sizes were decreased, which tend to zero at large sample sizes, so also the variances.

Key words: Autocorrelation function, autoregressive, mean variance, moving average, partial autocorrelation function

INTRODUCTION

Stationary time series process exhibit invariant properties over time with respect to the mean and variance of the series. Conversely, for non-stationary time series, the mean, variance, or both will change over the trajectory of the time series. Stationary time series have the advantage of representation by analytical models against which forecasts can be produced. Non stationary models through a process of differencing can be reduced to a stationary time series and are so open to analysis applied to stationary processes.[1] The most important methods for dealing with econometrics and time series data, in the case of model fitting includes autoregressive (AR) and moving average (MA) models. The basic assumption of these models is the stationarity that the data being fitted to them should be stationary.[2] AR and MA models are mathematical models of the persistence, or autocorrelation in a time series. The models are widely used in econometrics, hydrology, engineering, and other fields.

There are several possible reasons for fitting AR and MA models to data. Modeling can contribute to understanding the physical system by revealing something about the physical process that builds persistence into the series. The models can also be used to predict behavior of a time series or econometric data from past values. Such a prediction can be used as a baseline to evaluate the possible importance of other variables to the system. They are widely used for prediction of economic and Industrial time series. Another use of AR and MA models is simulation, in which synthetic series with the same persistence structure as an observed series can be generated. Simulations can be especially useful for established confidence intervals for statistics and estimated econometrics quantities.[3] The applications where simulation methods may be useful is extensive and include diverse disciplines such as manufacturing systems, flight simulation,

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construction, healthcare, military, and economics. Systems or processes that can be modeled through an underlying probability distribution are open to simulation through the Monte Carlo method.[4]

The basic assumption of time series models is the stationarity. Parameters behaviors of the stationarity models have been studied empirically.[5,6] Frequently in real world scenarios due to the complexity of the system under investigation it may not be possible to evaluate the systems behavior by applying analytical methods. This is because in many situations, it is very difficult to get data that follow the stationarity pattern, even if there is, it is very difficult to get the required number of replicates for the sample sizes of interest. Under such conditions an alternative approach to model such system is through creating a simulation. Succinctly, simulation method provides an alternative approach to studying system behavior through creating an artificial replication or imitation of the real world system. Based on this, this study therefore, considers simulation procedure to examine the characteristics of the parameters in stationarity of AR and MA models. The effect of changes in orders of the two models and different sample sizes, which has not been established in the literature was examined.

AR processes

An AR model is simply a linear regression of the current value of the series against one or more prior values of the series. The value of (p) is called the order of the AR model. AR models can be analyzed with one of various methods, including standard linear least squares techniques.

Assume that a current value of the series is linearly dependent on its previous value, with some error. Then, we could have the linear relationship

$$X_t = \propto_1 X_{t-1} + \propto_2 X_{t-2} + \ldots + \propto_p X_{t-p} + e_t \quad 2.1$$

Where, $\propto_1, \propto_2, \ldots \propto_p$ are AR parameters and e_t is a white noise process with zero mean and variance σ^2

AR processes are as their name suggests regressions on themselves. Specifically, a p^{th}-order AR process $\{Xt\}$ satisfies the equation 1. The current value of the series Y_t is a linear combination of the p most recent past values of itself plus an "innovation" term e_t that incorporates everything new in the series at time *t* that is not explained by the past values. Thus, for every *t*, we assume that et is independent of $X_{t-1}, X_{t-2}, X_{t-3}, \ldots$ Yule[7] carried out the original work on AR processes.

MA model

The general MA model can be given as follows;

$$X_t = \beta_1 e_{t-1} + \beta_2 e_{t-2} + \ldots + \beta_q e_{t-q} + e_t \quad 2.2$$

We call such a series a MA of order q and generally represented by MA(q). The terminology MA arises from the fact that X_t is obtained by applying the weight $\beta_1, \beta_2, \ldots, \beta_q$ to the variable $e_t, e_{t-1}, \ldots, e_{t-q}$. MA model.[8]

Autocorrelation function (ACF) and partial autocorrelation function (PACF)

After a time series has been stationarized by differencing, the next step in fitting a model is to determine whether AR or MA terms are needed to correct any autocorrelation that remains in the differenced series. There is a more systematic way to do this. By looking at the ACF and PACF plots of the differenced series known as correlogram, you can tentatively identify the numbers of AR and/or MA terms that are needed. The ACF plot: It is merely a bar chart of the coefficients of correlation between a time series and lags of itself. The PACF plot is a plot of the partial correlation coefficients between the series and lags of itself.[9] ACF represents the degree of persistence over respective lags of variable at the X_t and $X_t + {}_k$. PACF measures the amount of correlation between two variables which is not explained by their mutual correlation with a specified set of variables. Primary distinguishing characteristics of theoretical ACFs and PACFs for stationary processes is tabulated in Table 1:

MATERIALS AND METHODS

Data were simulated, under the assumption of stationary, from linear AR and MA processes

Table 1: Distinguishing characteristics of ACF and PACF for stationary process

Process ACF	PACF
AR Tails off toward to Zero (exponentially decay or damped sine wave)	Cutoff to Zero (After lag p)
MA cutoff to Zero (after lag q)	Tails off toward zero (exponentially decay).

of first, second, and third orders at different sample sizes. The forms of AR and MA processes considered for the simulation is given as follows;

i. $Y_t = \varnothing Y_{t-1} + e_t$

ii. $Y_t = \varnothing_1 Y_{t-1} + \varnothing_2 Y_{t-2} + e_t$

iii. $Y_t = e_t + \theta_1 e_{t-1}$

iv. $Y_t = e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2}$

The current value of the series is a linear combination of p most recent past values of itself plus an "innovation" term e_t that incorporates everything new in the series at time t that is not explained by the past values. Thus, for every t we assume that e_t is independent of Y_{t-1}, Y_{t-2}, Y_{t-3}.

Data simulated for both response variables and error terms from normal distribution with mean zero and variance one, that is,

$$Y_{ti} \sim N(0,1) \text{ and } e_{ti} \sim N(0,1)$$

$$t = 1,2,\ldots,20,40,60\ .\ .\ .\ .\ 200.\ i = 1,2,\ldots,1000$$

The normality of error term with zero mean and positive variance indicates that the error term is a white noise and therefore the data generated from these series is stationary. More so the parameter values were fixed for the models in such a way that it will exhibit a stationarity and did violate the stationarity conditions. Test of stationarity like ADF test was used to verify the stationarity status. Thereafter, different orders of autocorrelation (ρ_k) and partial autocorrelation ($\varnothing_{kk}$) values (where k =1, 2, 3,…, 10) were determined for every order simulated AR and MA models, sample sizes and parameter combinations. The effect of sample sizes n = 20, 60,…., 200 on the stationarity of the models was also studied. At every sample size, the stationary status of the p and q is, respectively, determined, where p, q = 1, 2. Parameters such as Mean, Variance, ACF, and PACF were determined.

DATA ANALYSIS

The results of simulations at various categories of sample sizes and parameters are presented accordingly in Table 2. The ACF and PACF were computed at different sample sizes and parameter combinations to determine their behavior on different levels of AR and MA. The results are presented in tables according to sample sizes and parameters.

Table 2 shows that the ACF and PACF values increase in absolute as sample parameter values is increased at sample size 50. However, as order of the AR increases, the ACF and PACF values decreases, respectively, with increases in parameter. Similarly, the increase in values of both ACF and PACF was observed, with increase in parameter for sample size 150 and 300 then decreases as order is getting large, except that, the decrease tend to zeros at both sample sizes for both correlation values. These exhibit the rules of stationarity of AR (1) of decrease in autocorrelation values as autocorrelation order increases [Table 3].

It is observed from Table 4 that both ACF and PACF increases as the second parameter increases but have close values from one parameter to another. Their values also decrease across autocorrelation orders and sample sizes. Both values of ACF and PACF tend to zero as the order increases.

Table 5 shows that the ACF and PACF values increase in absolute as sample parameter values is increased at sample size 50, 150, and 300. However, as order of the MA increases, the values of ACF and PACF decreases and tend toward zero at higher orders which follows the characteristics of MA. This is clearly observed in large sample size (n = 300) and their values decline as sample size increases when compare order to order of across the ample sizes.

Table 6 shows that the mean values of AR (1) increases with increase in parameter and decreases with increase in sample size which tend to zero at large sample size, so also the variances. However, for AR (2) and AR (3), both values of means and variances are close and almost equal across the parameter, especially at larger sample sizes. This indicates the stationarity of the AR models.

Table 7 also shows that the mean values of MA (1) increases with increase in parameter and decreases with increase in sample size which tend to zero at large sample size, so also the variances. However, for MA (2) and MA (3), both values of means and variances are close and almost equal across the parameter, especially at larger sample sizes. This indicates the stationarity of the AR models.

Table 2: ACF and PACF results for simulated data from AR (1) at sample sizes of 50, 150, and 300

K	ACF (N=50)				PACF (N=50)			
	∅=0.2	∅ =0.4	∅ =0.6	∅ =0.8	∅ =0.2	∅ =0.4	∅ =0.6	∅ =0.8
1	0.189	0.377	0.609	0.769	0.220	0.477	0.609	0.756
2	−0.004	−0.063	−0.102	−0.152	−0.007	−0.049	−0.105	−0.156
3	0.001	0.056	0.093	0.119	0.002	0.028	0.101	0.141
4	0.001	0.014	0.018	0.019	−0.002	−0.009	−0.029	−0.036
5	0.000	−0.008	−0.010	−0.017	−0.002	−0.009	0.010	0.028
6	0.000	−0.006	−0.014	−0.016	−0.003	−0.005	−0.009	−0.022
7	0.000	−0.006	−0.012	−0.014	−0.001	−0.004	−0.008	−0.015
8	0.000	0.002	0.0011	0.012	0.000	−0.001	−0.006	−0.009
9	0.000	0.002	0.006	0.008	0.000	0.003	0.003	0.004
10	0.000	−0.002	−0.002	−0.005	0.000	0.000	−0.001	−0.004
K	**ACF (N=150)**				**PACF (N=150)**			
	∅ =0.2	∅ =0.4	∅ =0.6	∅ =0.8	∅ =0.2	∅ =0.4	∅ =0.6	∅ =0.8
1	0.092	0.093	0.100	0.133	0.012	0.063	0.300	0.393
2	0.059	0.070	0.081	0.092	−0.010	−0.018	−0.113	−0.226
3	−0.028	−0.030	−0.031	−0.034	0.010	−0.012	0.015	0.046
4	−0.006	−0.011	−0.018	−0.028	−0.002	−0.004	−0.004	−0.006
5	0.001	0.010	0.011	0.014	0.002	0.003	0.004	0.009
6	0.001	0.010	0.010	0.012	0.000	−0.001	0.003	0.062
7	0.000	0.004	0.005	0.056	0.000	0.001	−0.002	−0.004
8	0.000	−0.004	0.004	0.029	0.000	0.000	0.003	0.004
9	0.000	0.000	−0.002	−0.028	0.000	0.000	−0.001	−0.001
10	0.000	0.000	0.000	−0.007	0.000	0.000	0.001	0.001
K	**ACF (N=300)**				**PACF (N=300)**			
	∅ =0.2	∅ =0.4	∅ =0.6	∅ =0.8	∅ =0.2	∅ =0.4	∅ =0.6	∅ =0.8
1	0.197	0.382	0.600	0.737	0.197	0.322	0.540	0.737
2	−0.016	−0.114	−0.360	−0.403	−0.171	−0.243	−0.317	−0.363
3	0.015	−0.103	−0.114	−0.209	0.083	0.143	0.224	0.286
4	0.010	0.035	0.107	0.108	−0.015	−0.054	−0.125	−0.199
5	−0.005	−0.022	−0.026	−0.067	−0.006	−0.012	0.037	0.101
6	−0.004	−0.012	−0.018	−0.055	0.005	−0.005	−0.014	−0.083
7	0.000	0.009	0.006	0.045	0.000	0.000	0.009	0.032
8	0.000	0.000	0.004	0.009	0.000	0.000	−0.002	−0.006
9	0.000	0.000	0.006	0.006	0.000	0.000	0.002	0.004
10	0.000	0.000	0.000	−0.002	0.000	0.000	−0.001	−0.131

Table 3: ACF and PACF results for a simulated from MA (1) at sample sizes of 50, 150, and 300

K	ACF (N=50)				PACF (N=50)			
	∅ =0.2	∅=0.4	∅=0.6	∅=0.8	∅=0.2	∅=0.4	∅=0.6	∅=0.8
1	0.294	0.505	0.682	0.914	0.294	0.505	0.682	0.914
2	0.116	0.205	0.466	0.830	0.032	−0.067	0.078	−0.085
3	0.027	0.229	0.330	0.754	−0.017	−0.020	0.022	0.033
4	−0.018	−0.043	0.619	0.675	−0.027	−0.050	−0.052	−0.061
5	0.025	−0.051	0.053	0.015	0.042	−0.057	−0.062	0.071
6	−0.018	−0.032	0.019	0.002	−0.014	−0.014	−0.117	−0.056
7	0.002	0.024	−0.003	0.009	0.006	0.009	0.064	−0.070
8	0.001	−0.005	−0.068	0.095	−0.016	−0.016	−0.017	−0039
9	0.020	0.055	0.059	0.067	0.011	0.094	0.111	0.183
10	0.007	0.072	−0.096	0.098	−0.033	−0.041	−0.057	−0.617

(Contd...)

Table 3: *(Continued)*

K	ACF (N=150)				PACF (N=150)			
	Ø=0.2	Ø=0.4	Ø=0.6	Ø=0.8	Ø=0.2	Ø=0.4	Ø=0.6	Ø=0.8
1	0.292	0.489	0.674	0.870	0.292	0.489	0.674	0.870
2	0.087	0.217	0.446	0.737	0.081	−0.089	−0.096	−0.098
3	0.067	0.142	0.313	0.616	0.078	0.082	0.093	−0.094
4	0.058	0.079	0.214	0.508	0.030	−0.033	−0.041	−0.061
5	0.013	0.024	0.108	0.420	−0.022	−0.024	−0.038	0.041
6	−0.006	0.040	0.068	0.329	−0.015	0.015	−0.019	−0.037
7	0.006	0.064	0.066	0.256	0.010	0.013	0.017	0.026
8	0.004	0.004	0.047	0.206	0.004	0.013	0.015	0.022
9	0.004	0.002	0.046	0.152	0.003	−0.007	0.007	−0.008
10	0.002	0.003	0.017	0.107	0.002	−0.008	−0.007	−0.006
K	ACF (N=300)				PACF (N=300)			
	Ø=0.2	Ø=0.4	Ø=0.6	Ø=0.8	Ø=0.2	Ø=0.4	Ø=0.6	Ø=0.8
1	0.159	0.362	0.569	0.747	0.159	0.362	0.569	0.747
2	0.043	0.059	0.059	0.066	0.018	0.032	0.042	0.049
3	−0.002	0.044	0.047	0.049	−0.012	−0.027	−0.028	−0.045
4	−0.002	0.015	0.118	0.284	0.012	0.003	0.008	−0.028
5	0.001	0.010	0.064	0.181	−0.002	0.007	−0.008	−0.032
6	0.000	0.003	0.033	0.105	0.001	−0.003	−0.004	−0.016
7	0.000	0.003	0.008	0.036	0.000	0.003	−0.013	−0.014
8	0.001	−0.002	−0.004	−0.024	0.000	0.000	−0.001	−0.009
9	0.000	0.000	−0.000	−0.005	0.000	0.000	0.000	0.003
10	0.000	0.000	−0.000	−0.001	0.000	0.000	0.000	−0.002

Table 4: ACF and PACF results for a simulated from AR (2) at sample sizes of 50, 150, and 300

K	ACF (N=50)				PACF (N=50)			
	Ø1=0.2 Ø2=0.2	Ø1=0.4 Ø2=0.4	Ø1=0.2 Ø2=0.4	Ø1=0.2 Ø2=0.6	Ø1=0.2 Ø2=0.2	Ø1=0.4 Ø2=0.4	Ø1=0.2 Ø2=0.4	Ø1=0.2 Ø2=0.6
1	0.164	0.310	0.164	0.168	0.164	0.330	0.169	0.172
2	−0.159	−0.246	−0.161	−0.168	−0.101	−0.158	−0.101	−0.113
3	−0.152	−0.161	−0.158	−0.159	0.091	0.135	0.098	0.098
4	0.104	0.136	0.121	0.124	0.087	0.170	0.087	0.089
5	0.083	0.055	0.087	0.089	0.041	−0.167	0.041	0.041
6	−0.077	−0.136	−0.079	−0.079	−0.035	−0.131	−0.035	−0.035
7	−0.007	−0.006	−0.006	−0.007	0.008	0.009	0.009	0.008
8	0.005	0.006	0.005	0.006	0.003	0.002	0.005	0.008
9	0.003	0.004	0.003	0.003	0.005	−0.003	0.005	0.005
10	−0.001	−0.001	−0.002	−0.001	−0.004	−0.002	−0.003	−0.004
K	ACF (N=150)				PACF (N=150)			
	$Ø_1$=0.2 $Ø_2$=0.2	$Ø_1$=0.4 $Ø_2$=0.4	$Ø_1$=0.2 $Ø_2$=0.4	$Ø_1$=0.2 $Ø_2$=0.6	$Ø_1$=0.2 $Ø_2$=0.2	$Ø_1$=0.4 $Ø_2$=0.4	$Ø_1$=0.2 $Ø_2$=0.4	$Ø_1$=0.2 $Ø_2$=0.6
1	0.114	0.313	0.164	0.176	0.103	0.310	0.106	0.118
2	0.105	0.258	0.155	0.163	0.101	0.177	0.101	0.111
3	0.104	0.109	0.104	0.107	−0.104	−0.105	−0.104	−0.108
4	0.082	0.109	0.084	0.101	0.092	0.102	0.097	0.102
5	−0.076	−0.080	−0.076	−0.070	−0.092	−0.075	−0.094	−0.098
6	−0.066	−0.068	−0.066	−0.068	−0.090	−0.064	−0.090	−0.091
7	−0.006	−0.006	−0.006	−0.006	−0.003	0.003	−0.003	−0.003
8	0.005	0.005	0.005	0.005	0.002	0.002	0.002	0.002
9	−0.001	−0.005	−0.006	−0.007	−0.002	−0.002	−0.002	−0.005
10	0.000	0.005	0.000	0.006	0.000	0.000	0.002	0.004

(Contd...)

Table 4: *(Continued)*

K	ACF (300)				PACF (300)			
	$\emptyset_1=0.2$ $\emptyset_2=0.2$	$\emptyset=0.4$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.6$	$\emptyset_1=0.2$ $\emptyset_2=0.2$	$\emptyset_1=0.4$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.6$
1	0.098	0.116	0.109	0.112	0.098	0.126	0.108	0.120
2	0.090	0.105	0.090	0.100	0.072	0.102	0.092	0.094
3	0.065	0.080	0.075	0.091	−0.035	−0.101	−0.041	−0.055
4	−0.050	0.076	−0.072	−0.078	−0.034	−0.102	−0.038	−0.054
5	−0.047	−0.056	−0.049	−0.057	−0.023	−0.057	−0.024	−0.043
6	−0.003	−0.003	−0.002	−0.002	−0.002	0.003	−0.003	−0.003
7	−0.002	−0.004	−0.002	−0.002	−0.002	−0.003	−0.002	−0.002
8	−0.003	−0.002	−0.003	−0.003	0.003	0.002	0.001	0.002
9	0.000	0.002	0.000	0.006	0.000	0.003	0.006	0.006
10	0.000	−0.001	0.000	−0.002	0.000	−0.002	−0.001	−0.001

Table 5: ACF and PACF results for a simulated from MA (2) at sample sizes of 50, 150, and 300

K	ACF (N=50)				PACF (N=50)			
	$\emptyset_1=0.2$ $\emptyset_2=0.2$	$\emptyset_1=0.4$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.6$	$\emptyset_1=0.2$ $\emptyset_2=0.2$	$\emptyset_1=0.4$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.6$
1	0.247	0.367	0.447	0.547	0.247	0.367	0.447	0.547
2	−0.116	−0.124	−0.136	−0.136	−0.189	−0.298	−0.389	−0.389
3	−0.085	−0.086	−0.085	−0.085	−0.005	0.108	−0.005	−0.005
4	0.078	0.087	0.088	0.078	0.091	0.108	0.091	0.091
5	−0.055	−0.044	−0.055	−0.055	−0.043	−0.040	−0.043	−0.043
6	−0.009	−0.009	−0.009	−0.009	0.005	0.006	0.005	0.005
7	0.008	0.005	0.008	0.008	0.004	−0.002	0.004	0.004
8	−0.004	−0.005	−0.004	−0.004	−0.002	−0.002	−0.002	−0.002
9	−0.002	−0.003	−0.002	−0.002	0.007	0.002	0.007	0.007
10	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001

K	ACF (150)				PACF (300)			
	$\emptyset_1=0.2$ $\emptyset_2=0.2$	$\emptyset_1=0.4$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.6$	$\emptyset_1=0.2$ $\emptyset_2=0.2$	$\emptyset_1=0.4$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.6$
1	0.080	0.231	0.380	0.490	0.080	0.231	0.380	0.490
2	−0.081	−0.084	−0.081	−0.081	−0.088	−0.145	−0.088	−0.088
3	0.069	0.040	0.069	0.069	0.084	0.104	0.084	0.084
4	−0.113	−0.112	−0.113	−0.113	−0.137	−0.176	−0.137	−0.137
5	−0.101	−0.132	−0.101	−0.101	−0.065	−0.042	−0.065	−0.065
6	−0.013	0.024	−0.013	−0.013	−0.028	0.035	−0.028	−0.028
7	0.005	0.002	0.005	0.005	0.007	0.006	0.007	0.007
8	0.003	0.001	0.001	0.001	0.008	−0.008	0.007	0.007
9	−0.003	−0.001	−0.004	−0.004	−0.004	0.002	−0.004	−0.004
10	0.001	0.000	0.001	0.001	0.001	0.001	−0.001	−0.001

K	ACF				PACF			
	$\emptyset_1=0.2$ $\emptyset_2=0.2$	$\emptyset_1=0.4$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.6$	$\emptyset_1=0.2$ $\emptyset_2=0.2$	$\emptyset_1=0.4$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.4$	$\emptyset_1=0.2$ $\emptyset_2=0.6$
1	0.201	0.350	0.201	0.201	0.201	0.350	0.201	0.201
2	0.003	0.013	0.003	0.003	−0.039	−0.125	−0.039	−0.039
3	0.056	0.058	0.056	0.056	0.066	0.112	0.066	0.066
4	0.027	0.029	0.027	0.027	0.002	−0.037	0.002	0.002
5	−0.022	−0.006	−0.022	−0.022	−0.026	0.004	−0.026	−0.026
6	0.013	0.019	0.013	0.013	0.015	0.014	0.015	0.015
7	0.002	0.005	0.002	0.002	0.005	0.003	0.005	0.005
8	−0.003	−0.002	−0.003	−0.003	−0.006	−0.005	−0.006	−0.006
9	−0.001	−0.001	−0.001	−0.001	0.000	0.001	0.001	0.000
10	0.001	−0.001	0.001	0.001	0.001	−0.001	0.001	0.001

Table 6: Means and variances of AR(1) and AR (2) at different sample sizes

Sample size (n)	Mean				Variance			
	∅=0.2	∅=0.4	∅=0.6	∅=0.8	∅=0.2	∅=0.4	∅=0.6	∅=0.8
50	−0.6370	−07923	−0.9165	−1.2828	0.7378	1.0830	2.0317	6.6340
150	0.1610	0.2383	0.3287	0.4111	0.6850	1.0346	1.5198	3.2761
300	−0.0295	−0.0240	−0.0352	0.0164	0.1798	1.0258	1.7142	2.6618
Sample size (n)	**Mean**				**Variance**			
	$∅_1$=0.2 $∅_2$=0.2	$∅_1$=0.4 $∅_2$=0.4	$∅_1$=0.2 $∅_2$=0.4	$∅_1$=0.2 $∅_2$=0.6	$∅_1$=0.2 $∅_2$=0.2	$∅_1$=0.4 $∅_2$=0.4	$∅_1$=0.2 $∅_2$=0.4	$∅_1$=0.2 $∅_2$=0.6
50	0.1963	0.0840	0.1963	0.1963	1.5781	1.6635	1.5781	1.5781
150	0.0433	0.0831	0.0433	0.0433	1.0208	1.1517	1.0208	1.0208
300	0.0148	0.0294	0.0148	0.0148	1.1511	1.3471	1.1511	1.1511

Table 7: Means and Variances of MA (1) and MA (2) at Different Sample Sizes

Sample size (n)	Mean				Variance			
	∅=0.2	∅=0.4	∅=0.6	∅=0.8	∅=0.2	∅=0.4	∅=0.6	∅=0.8
50	−0.2370	−0.2834	−0.3298	−0.3762	1.2313	1.3271	1.5173	1.8018
150	0.0609	0.0709	0.0810	0.0910	1.0747	1.2816	1.5642	1.9224
300	−0.0127	−0.0148	−0.0170	−0.0191	1.0676	1.2023	1.4183	1.7153
Sample size (n)	**Mean**				**Variance**			
	$∅_1$=0.2 $∅_2$=0.2	$∅_1$=0.4 $∅_2$=0.4	$∅_1$=0.2 $∅_2$=0.4	$∅_1$=0.2 $∅_2$=0.6	$∅_1$=0.2 $∅_2$=0.2	$∅_1$=0.4 $∅_2$=0.4	$∅_1$=0.2 $∅_2$=0.4	$∅_1$=0.2 $∅_2$=0.6
50	0.4131	0.4762	0.4131	0.4131	0.8801	1.0150	0.8801	0.8801
150	0.0233	0.0290	0.0233	0.0233	1.3459	1.4531	1.3459	1.3459
300	−0.0180	−0.0209	−0.0180	−0.0180	1.0198	1.1412	1.0198	1.0198

CONCLUSIONS

The study concluded that the absolute values of AR and MA models increase as the parameter values increase but decrease with increases of AR and MA models and tends to zero at higher lag orders. This is clearly observed in large sample size (n = 300) and their values decline as sample size increases when compare order to order of across the ample sizes. That is, the values of ACF and PACF are getting smaller in absolute values as sample size getting higher. This implies that zero values of both ACF and PACF are observed at smaller lag when sample size is large at all parameters.

Furthermore, it is observed that both ACF and PACF increase in AR and MA of orders 2 and 3, as the second parameter increases but have close values from one parameter to another. Their values also decrease across autocorrelation orders and sample sizes. Both values of ACF and PACF tend to zero as the lag order increases. Their values increase in absolute as sample parameter values is increased at sample size 50, 150, and 300. However, as order of the MA increases, the values of ACF and PACF decreases and tend toward zero at higher orders and their values decline as sample size increases when compare lag order to order of across the ample sizes. Finally, it was examined that the mean values of the AR and MA models of first-order increases with increase in parameter and decreases with increase in sample size which tend to zero at large sample size, so also the variances. However, for second and third orders of AR and MA, both values of means and variances are close and almost equal across the parameter, especially at larger sample sizes. This indicates the stationarity of the AR models.

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