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RESEARCH ARTICLE

Exploring the Methods of Cointegration Procedures in Dynamics of HIV/AIDS and Related Infections

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ABSTRACT

Introduction: Cointegration has become an important property in contemporary time series analysis. Time series often have trends - either deterministic or stochastic. **Material and Methods:** This research work seeks to determine the inherent long run relationship between the human immunodeficiency virus (HIV) and other diseases and the circumstances when it is reasonable to expect that two or more diseases may be cointegrated. That is, if at least one of the processes is driving the other and if the diseases are being driven by the same underlying process. **Result:** The data (Prevalence of HIV/Acquired Immunodeficiency Syndrome [AIDS] Incidences) modeled in this study were obtained from the National Agency for the Control of AIDS. The stationarity characteristics of the study variables were investigated using Augmented Dickey-Fuller test, the long-run relationship between HIV and other two diseases was determined using Engle-Granger cointegration, Phillips-Ouliaris, and Johansen testing procedure while the Granger causality test was also performed to determine the short run relationship of the variables. **Conclusion:** Results showed that the series are integrated of order two; HIV, Hepatitis, and TB are found to be strongly and significantly positively correlated, the data series are considered to be stationary after the second differences. Furthermore, the Granger causality tests show that HIV "Granger causes" Tuberculosis and Hepatitis in Nigeria. However, Phillips-Ouliaris Test for Cointegration Determines the strongest cointegration level among HIV/AIDS and other infections. Hence, it is the most robust test for testing cointegration between HIV and tuberculosis, and HIV and Hepatitis disease.

Key words: Cointegration, Unit root, Causality, Human immunodeficiency virus and acquired immunodeficiency syndrome

INTRODUCTION

The concept cointegration in the time-series econometrics was introduced by Granger (1981)[1] and Engle and Granger (1987)[2] and they provided a theoretical frameworks for representing, testing, estimating and modeling of cointegrated non-stationary time-series variables. Ever since, the concept has undergone various developments and transformations from researchers such as Utkulu (1994)[3] and Alexander (1999).[4] By cointegration analysis a non-stationary data can used such that spurious results are avoided, and also provides effective framework for testing and estimating long-run models from time-series data.

Cointegration is important in time series data that involve more than one variable due to the fact that if relationship between two variables holds, it should be possible to predict one variable from another. That is, if markets move together in the long-run, this hypothesis will hold (Akeyede *et al*., 2018).[5] Cointegration is a statistical property of a collection (X_1, X_2., X_k) of time series variables. First, all of the series must be integrated of order d (see Order of integration). Next, if a linear combination of this collection is integrated of order less than d, then the collection is said to be co-integrated. Formally, if (*X*, *Y*, and *Z*) are each integrated of order d, and there exist coefficients a, b, and c such that aX + bY + cZ is integrated of

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order less than d, then X, Y, and Z are cointegrated. If two or more series are individually integrated (in the time series sense) but some linear combination of them has a lower order of integration, then the series are said to be co-integrated. A common example is where the individual series are first-order integrated [I(1)] but some (cointegrating) vector of coefficients exists to form a stationary linear combination of them. For instance, a stock market index and the price of its associated futures contract move through time, each roughly following a random walk. Testing the hypothesis that there is a statistically significant connection between the futures price and the spot price could now be done by testing for the existence of a cointegrated combination of the two series. Cointegration has evolved into is a time-series modeling methodology and Alexander (1999)[4] discussed in details, the three popular techniques of measuring cointegration which are Engle-Granger (EG) estimation procedure; the Phillip-Ouliaris residual-based test; and Johansen's multivariate technique.

Human immunodeficiency virus and acquired immunodeficiency syndrome (HIV/AIDS) in recent epidemiological data indicate that it remains a public health issue that persistently drains our economic sector having claimed more than 25 million lives over the last three decades (UNAID 2013).[6] The spread of the infection over the past 30 years has a great impact on health, welfare, employment and criminal justice sectors; affecting all social and ethnic groups throughout the world. It is highly noticed in Nigeria especially among tertiary students. HIV infections have been recognized as one of the most worrisome killer diseases in the world today. Daniel *et al*. (2013)[7] investigate the time series analysis of HIV/AIDS carriers using least square method and autoregressive model. The result showed that AR is selected as the best model for modeling HIV/AIDS carriers. The result further indicated that the prevalence rate of HIV/AIDS carriers fluctuates over the period "t."

Akeyede *et al*. (2018)[5] under-seek time series cross-correlation analysis of HIV seropositivity using Estimated Cross-Correlation function. The result indicated that there is significant positive correlation between the proportion of HIV seropositive migrants and the proportion of pulmonary TB cases among migrant. Long *et al*. (2018)[8] seek to predict new HIV cases thus: Time Series Forecast of New HIV Cases in Ashanti Region of Ghana. Using Holt's exponential smoothing and the Box Jenkins ARIMA model of time series analysis, 2580 new HIV cases are predicted per year in Ashanti region whereas the Box-Jenkins ARIMA model predicted a constant number of 2556 new HIV cases per year. Demissew Tsigemelak Roeger (2009)[9] worked on Modeling and Projection of HIV/AIDS epidemics in Ethiopia using ARIMA. The result that the trend of HIV/AIDS prevalence was increasing in alarming rate from mid-1990's and reached its climax in the years 2002 to 2004 and decreased onward. The Autoregressive Integrated Moving Average (ARIMA) time series analysis model and ARIMA (2, 3, 2) was best fit for the observed data. Akeyede *et al*. (2018)[5] discussed on a cointegration and causality analysis of HIV/AIDS and Some Opportunities Infections in Nigeria and found out that the data series are integrated of order one, HIV and TB are found to be strongly and significantly positively correlated, the data sets of the states except HIV cases from Niger are considered to be stationary after the first differenced.

This paper investigated three cointegration measuring methodologies on HIV and some related diseases data. The research work also seeks to determine the inherent long run relationship between the HIV and other diseases and the circumstances when it is reasonable to expect that two or more diseases may be cointegrated. That is, if at least one of the processes is driving the other and if the diseases are being driven by the same underlying process.

METHODOLOGY

The data (Prevalence of HIV/AIDS Incidences) modeled in this study were obtained from National Agency for the Control of AIDS. This study presents respective prevalence rate, the yearly series ranges from 1995 to 2020 amounting to 24 observations. The data was presented with a time series plot so as to check if the series follows a trend or seasonality. Augmented Dickey-Fuller Test was used to test the null hypothesis; that the data needs to be differenced to make it stationary versus the alternative hypothesis that the data is stationary and does not need to be differenced. Furthermore, a cointegration test was carried out among the HIV/AIDS cases and related infections

(Tuberculosis and Hepatitis). Thereafter, the causality analysis was carried out on the data.

Test of Stationarity using Augmented Dickey-Fuller (ADF) Test

The ADF test is used to test for stationarity/unit root. The testing procedure for the ADF test is the same as for the Dickey-Fuller test but it is applied to the model (1).

$$\Delta Y_t = \alpha + \beta_t + \Upsilon Y_{t-1} + \sigma_1 \Delta Y_{t-1} + . + \sigma p\text{-}_1 \Delta Y_{t-p+1} + e_t \quad (1)$$

Where α is a constant, β the coefficient on a time trend, and P the lag order of the autoregressive process. Imposing the constraints $\alpha = 0$ and $\beta = 0$ corresponds to modeling a random walk and using the walk with a drift.

The test statistic, value is calculated as follows:

$$t = \frac{\hat{Y}}{\sigma_{\hat{Y}}} \quad (2)$$

where $\hat{Y}$ is the estimated coefficient and $\sigma_{\hat{Y}}$ is the standard error in the coefficient estimate.

The null-hypothesis for an ADF test: H_0: $\Upsilon = 1$Vs H_1: $\Upsilon < 1$.

Where H_0: Is the null hypothesis (has unit root) and H_1: Does not have unit root. The test statistic value t is compared to the relevant critical value for the Dickey-Fuller test. If the test statistic is less than the critical value, we reject the null hypothesis and conclude that no unit-root is present. The ADF test does not directly test for stationarity but indirectly through the existence (or absence) of a unit-root. Using the usual 5% threshold, differencing is required if the $P > 0.05$.

Concept of Cointegration

The main idea behind cointegration is that variables have a tendency to move together in the long run, there is an equilibrium relationship between them. Short-term deviations from the equilibrium are possible, but in the long-run the variables will return back to equilibrium relation due to the error or equilibrium correction model (Engle and Granger, 1987).[2]

The concept of cointegration has its roots in the work of Engle and Granger (1987).[2] Two variables are cointegrated if they share a common stochastic trend in the long-run. The general rule when combining two integrated variables is that their combination will always be integrated at the higher of the two orders of integration. The most common order of integration in time series is either zero or one (Brooks, 2008);[10]

i. if $y_t \sim I(0)$, and $xt \sim I(0)$, then their combination $(ax_t + by_t)$ will also be I(0),

ii.if $y_t \sim I(0)$, and $x_t \sim I(1)$, then their combination $(ax_t + by_t)$ will now be I(1), because I(1) is higher order of integration and dominates the lower order of integration I(0),

iii.if $y_t \sim I(1)$, and $x_t \sim I(1)$, then their combination $(ax_t + by_t)$ will also be I(1), in the general case.

However, if there exists such linear combination of non-stationary variables I(1) that is stationary, I(0), cointegration between those variables exists. The following regression model includes two I(1) non-stationary variables y_t and x_t:

$$y_t = \mu + \beta x_t + e_t \quad (3)$$

If the OLS estimate is such that the linear combination of y_t and x_t stationary, these two variables are cointegrated. The error term between them is then constant over time (stationary):

$$e_t = y_t - \beta x_t \quad (4)$$

For two variables to be cointegrated they need to be integrated of the same order. For example if one variable is I(0) and the other one is I(1), they cannot be cointegrated. The highest order of integration of the two variables will dominate and cointegration will not exist. (Bollerslev, Chou and Kroner, 1992).[11] However, if there is a linear combination of the stock indices that is stationary, cointegration between them exists. The EG single-equation method is applied to perform pairwise analysis of the stock indices presented in chapter 4. It allows only for one endogenous and one exogenous variable.

Granger Causality Test

Cointegration indicates existence of a long-run relationship between variables. Even when the variables are not cointegrated in the long-run, they might still be related in the short-run. In order to understand short-run interdependence among stock markets, Granger causality tests will be performed. Granger causality test is based on a standard F-test which seeks to determine if changes in one variable cause changes in another variable. A variable X is said to "Granger cause" variable Y, if the previous values of X could

predict the current value of Y. Let us start with a simple VAR model:

$$y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \ldots + \beta_k y_{t-k} + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \ldots + \alpha_k x_{t-k} + e_t \quad (5)$$

If all α coefficients on lagged values of X are significant in this equation, then "X Granger causes Y." If X Granger causes Y and not vice versa, it is called unidirectional causality. If the causality goes both ways from X to Y and from Y to X, then this is called bidirectional causality (Brooks, 2008).[10]

After estimating the VAR, restrictions are imposed and the following hypotheses are tested in a Granger causality test:

$H_0 : \alpha_1 = \alpha_2 = \ldots = \alpha_k = 0$ ("X does not Granger causeY")

$H_1 : \alpha_1 \neq \alpha_2 \neq \ldots \neq \alpha_k \neq 0$, for at least one of α_i coefficients ("X does not Granger causeY")

The test statistic follows a F distribution, with *p* degrees of freedom under the null hypothesis. *P* is the optimal number of lags.

The term "causality" should not be wrongly interpreted - it does not mean that changes in one variable cause changes in the other variable. It simply means that there is a correlation between the current value of one variable and the previous values of another variable. We used Granger causality tests to examine the lead-lag relationships among diseases across the states.

However, these tests can only provide information of whether a significant impact exists between diseases, but nothing about the sign of the impact or how long it will last. An impulse response analysis could give us answers regarding this, but the cointegration between the diseases is the focus of this study.

The EG Test

The EG test is a single-equation method used to determine whether there is a cointegrating relationship between two variables (Engle and Granger, 1987).[11] The precondition to examine cointegration is that the variables are both non-stationary and integrated of the same order. The EG method can be performed by following the next four step procedure:

Step 1: Perform the ADF test as demonstrated in chapter 4 to pretest for the order of integration. If the variables are both I(1), cointegration is theoretically possible and we can proceed to step 2. If the variables are of different order, the conclusion is that cointegration is not possible.

Step 2: Estimate the long-run, static relationship or equilibrium by running the OLS regression on the equation (3). This equation can be expanded with a constant term and a time trend, If the variables are cointegrated, an OLS regression will give a "super-consistent" estimator, denoted as $\hat{\beta}$, implying that the coefficient β will converge faster to its true value than using OLS on stationary variables, I(0) (Dolado *et al*., 1990).[12] If there is a linear combination of variables y_t and X_t that is stationary, the variables are said to be cointegrated. This linear combination of the variables can then be presented with the estimated error term in (4)

Step 3: Store the residuals and examine whether they are stationary or not. Here an ADF test, as explained earlier, is performed on the saved residuals from every regression equation above. The hypotheses for the EG test for cointegration are:

$H_0 : \hat{e}_t - I(1)$-non-stationary residual and nocointegration between variables

$H_1 : \hat{e}_t - I(0)$-stationary residual and cointegration between variables

If the null hypothesis is rejected, the variables from the model are cointegrated. The test statistics is the same as the one used for the ADF test, but the critical values are different. Since the EG method includes testing on estimated residuals ($\hat{e}_t$) instead of the actual values, the estimation error will change the distribution of the test statistics. Therefore, the critical values used in an EG approach will be larger in absolute value, or more negative compared to those used in a DF or ADF test. This means that the magnitude of the test statistics must be much larger to reject the null hypothesis, compared to the usual DF critical values. Davidson and Mackinnon (2001)[13] provides appropriate critical values for residual-based cointegration testing, depending on whether and which deterministic terms are included in the model.

Step 4: If cointegration is found between the variables, estimate an error-correction model. However, this will not be part of our analysis, since we are interested only in detecting cointegration.

Johansen Test

The Johansen test is a test for cointegration that allows for more than one cointegrating relationship, unlike the EG method, but this test

is subject to asymptotic properties, that is, large samples. If the sample size is too small, then the results will not be reliable and one should use Auto Regressive Distributed Lags.

Phillips-Ouliaris Cointegration Test

Phillips and Ouliaris (1990) show that residual-based unit root tests applied to the estimated cointegrating residuals do not have the usual Dickey-Fuller distributions under the null hypothesis of no-cointegration. Because of the spurious regression phenomenon under the null hypothesis, the distribution of these tests have asymptotic distributions that depend on (1) the number of deterministic trend terms and (2) the number of variables with which co-integration is being tested. These distributions are known as Phillips-Ouliaris distributions and critical values have been tabulated. In finite samples, a superior alternative to the use of these asymptotic critical value is to generate critical values from simulations.[14]

Granger Causality Test

Cointegration indicates existence of a long-run relationship between variables. Even when the variables are not cointegrated in the long-run, they might still be related in the short-run. In order to understand short-run interdependence among stock markets, Granger causality tests were performed. Granger causality test is based on a standard F-test which seeks to determine if changes in one variable cause changes in another variable. A variable X is said to "Granger cause" variable Y, if the previous values of X could predict the current value of Y. Let us start with a simple VAR model:

$$y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \ldots + \beta_k y_{t-k}$$
$$+\alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \ldots + \alpha_k x_{t-k} + e_t$$

If all α coefficients on lagged values of X are significant in this equation, then "X Granger causes Y." If X Granger causes Y and not vice versa, it is called unidirectional causality. If the causality goes both ways from X to Y and from Y to X, then this is called bidirectional causality (Brooks, 2008).

After estimating the VAR, restrictions are imposed and the following hypotheses are tested in a Granger causality test:

H_0: $\alpha_1 = \alpha_2 = = \alpha_K = 0$ ("X does not Granger cause Y")

H_1: $\alpha_1 \neq \alpha_2 \neq \neq \alpha_K \neq 0$, for at least one of α_i coefficients ("X does not Granger cause Y")

The test statistic follows a F distribution, with p degrees of freedom under the null hypothesis. p is the optimal number of lags.

The term "causality" should not be wrongly interpreted - it does not mean that changes in one variable cause changes in the other variable. It simply means that there is a correlation between the current value of one variable and the previous values of another variable. We used Granger causality tests to examine the lead-lag relationships among diseases across the states.

However, these tests can only provide information of whether a significant impact exists between diseases, but nothing about the sign of the impact or how long it will last. An impulse response analysis could give us answers regarding this, but the cointegration between the diseases is the focus of this study.

DATA ANALYSIS

Data analyses of HIV and related infections are presented in tables as follows:

Tables 1,2 and 3 above present ADF test for the HIV, tuberculosis, and hepatitis diseases, respectively, for the real data and after the data have been differenced for the 1st time. ADF statistic values of the two cases with *P*-values which are greater than the critical value of 0.05, indicates non rejection of the null hypothesis of having a unit root series and therefore conclude that that the data series of both states considered, are not stationary, they are indeed have unit roots. In conclusion, it is clear that the series has to be transformed or differenced to stabilize or stationaries the data before its capability is assessed or improvements are initiated, since the tests confer non stationarity in the data series.

Table 3 shows the ADF test for the second differenced, the three diseases in this case have *P*-values which are <5% level of significance indicate that, the null hypothesis of having a unit root series should be rejected in favor of alternative of being stationary. Therefore, the data series are considered to be stationary after the second differenced of tuberculosis HIV/AIDS and hepatitis data.

The results from Table 4 indicate cointegration between HIV, tuberculosis, and hepatitis at 1%

Table 1: Unit root tests for the data series

State	Values	Lag Order	*P*-value	Hypothesis (H_0)	Decision	Remarks
HIV	−2.9761	2	0.1998	Unit root	accept H_0	Not stationary
Tuberculosis	−2.1418	2	0.5183	Unit root	accept H_0	Not stationary
Hepatitis	−3.2504	2	0.09821	Unit root	accept H_0	Not stationary

Table 2: Unit root tests for the data series after first differenced

State	Values	Lag order	*P*-value	Hypothesis (H_0)	Decision	Remarks
HIV	−0.2633	2	0.9851	Unit root	Accept H_0	Not stationary
Tuberculosis	−2.3317	2	0.4460	Unit root	Accept H_0	Not stationary
Hepatitis	−1.7573	2	0.6648	Unit root	Accept H_0	Not stationary

Table 3: Unit root test of second differenced of data series

State	Values	Lag order	*P*-value	Hypothesis (H_0)	Decision	Remarks
HIV	−4.324	2	0.01195	Unit root	Reject H_0	Stationary
Tuberculosis	−2.5291	2	0.0370	Unit root	Reject H_0	Stationary
Hepatitis	−5.0154	2	0.0100	Unit root	Reject H_0	Stationary

Table 4: Regressions and engle-granger test for cointegration (HIV, and tuberculosis and hepatitis)

State	Lag order	DF	Coefficient β_i	Test statistics	*P*-value	Adjusted R-squared	Remarks
Tuberculosis	2	23	62.84	0.6239	2.827e-05	0.5206	Cointegrated
Hepatitis	1	23	29.20	1.0118	2.286e-08	0.7386	Cointegrated

Table 5: Regressions and Johansen test for cointegration (HIV and hepatitis)

State	Lag order	DF	Coefficient β_i	Test statistics	*P*-value	Adjusted R-squared	Remarks
Tuberculosis	2	26	14.571	1.3476	1.571e-05	0.646	cointegrated
Hepatitis	1	26	3.6621	0.8766	1.602e-06	0.5784	cointegrated

Table 6: Regressions and phillips–ouliaris test for cointegration (HIV, and tuberculosis and hepatitis)

State	Lag Order	DF	Coefficient β_i	Test statistics	*P*-value	Adjusted R-squared	Remarks
Tuberculosis	2	23	68.09	0.6239	1.007e-03	0.5206	cointegrated
Hepatitis	1	23	49.67	1.0118	1.006e-03	0.7386	cointegrated

significance level. The coefficient estimate β_i indicates that if the HIV increase by 1%, then the tuberculosis will increase by 62.84% and hepatitis 29.20%. The null hypothesis of no cointegration is rejected at 1% significance level, which is a very strong proof of cointegration between HIV and the related infections. The result from Table 5 indicates cointegration between HIV, tuberculosis, and hepatitis at 1% significance level. The coefficient estimate β_i indicates that if the HIV increase by 1%, then the tuberculosis will increase by 14.57% and hepatitis 3.66%. The null hypothesis of no cointegration is rejected at 1% significance level, which is a very strong proof of cointegration.

The results from Table 6 indicate cointegration between HIV, tuberculosis, and hepatitis at 1% significance level while Table 7 displays Granger causality test among HIV, tuberculosis, and hepatitis at 1% significance level. The coefficient estimate β_i indicates that if the HIV increase by 1%, then the tuberculosis will increase by 68.09%, and hepatitis 49.67%. The null hypothesis of no cointegration is rejected only at 1% significance level, which is a very strong proof of cointegration between HIV and others.

Table 7: Granger causality test among HIV, tuberculosis, and hepatitis

Null hypothesis	df	F-test statistic	*P*-value	Conclusion
HIV does not granger cause tuberculosis	17	0.9988	0.0090	Reject H_o
HIV does not granger cause hepatitis	17	0.9980	0.0126	Reject H_o

CONCLUSION

This study has tried to model the dynamics of HIV/AIDS and co-infection in Nigeria. The

stationarity characteristics of the study variables were investigated using ADF test, the long-run relationship between HIV and other two diseases was determined using, EG cointegration, Phillips-Ouliaris and Johansen testing procedure while the Granger causality test was also performed to determine the short run relationship of the variables. Results showed that the series are integrated of order two; HIV, Hepatitis, and TB are found to be strongly and significantly positively correlated, the data series are considered to be stationary after the second differenced.

Furthermore, the Granger causality tests shows that HIV "Granger causes" tuberculosis and hepatitis in Nigeria. However, Phillips-Ouliaris Test for Cointegration determines the strongest cointegration level among HIV/AIDS and other infections. Hence, it is most robust test for testing cointregration between HIV and tuberculosis, and HIV and hepatitis disease.

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