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RESEARCH ARTICLE

ALGEBRAIC SOLUTION OF FERMAT'S THEOREM (MATHEMATICS, NUMBER THEORY)

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ABSTRACT

Fermat's Last Theorem (or Fermat's last theorem) is one of the most popular theorems in mathematics. Formulated in French mathematician Pierre Fermat in 1637. Despite the simplicity of the formulation, literally, at the "school" arithmetic level, proof of the theorem sought by many mathematicians for more than three hundred years. And only in 1994 year the theorem was proven by the English mathematician Andrew Wilson with colleagues; the proof was published in 1995. With this article, the author completes his research on the given topic, makes corrections and eliminates the errors of the previous ones.

Keywords: Theorem, Fermat, elementary, solution.

INTRODUCTION

$$X^n + Y^n = Z^n \tag{01}$$

Where;

n- prime number, n>2; X,Y,Z are integers.
The solutions of which can be X, Y, Z - relatively prime numbers.

1.Decomposition of (01) into multipliers.

If n is odd, then (01) will decompose into multipliers:

$$X^n + Y^n = (X+Y)(X^{n-1} - X^{n-2}Y + ... - XY^{n-2} + Y^{n-1}) \tag{02}$$

where in the second bracket is the geometric progressionfirst term $a_1 = X^{n-1}$,and a multiplier $q = -\frac{Y}{X}$

The sum of the members of which $S = \frac{a_1(1-q^n)}{1-q}$

$$Z^n = Z_{11}\, Z_{22} \tag{03}$$

where

$$Z_{11}=X+Y \tag{04}$$

$$Z_{22}=X^{n-1}-X^{n-2}Y+...-X\ Y^{n-1}+Y^{n-1} \tag{05}$$

Equivalent representation Z_{22}

If we sum the equidistant terms from the middle term of the progression in pairs of the middle term of the progression in pairs we have:
For degree 3

$$Z_{22}=(X+Y)^2-3XY \tag{06}$$

Fifth degree:

$$Z_{22}=\frac{X^5+Y^5}{X+Y}=X^4-X^3\ Y+X^2\ Y^2-X\ Y^3+\ Y^4 \tag{07}$$

$$X^4+Y^4=(X+Y)^4-4\ X\ Y\ (X+Y)^2+2\ X^2\ Y^2 \tag{08}$$

$$-X\ Y^3-X^3\ Y=-X\ Y\ (X^2+Y^2)=-X\ Y(X+Y)^2+2\ X^2\ Y^2 \tag{09}$$

$$Z_{225}=(X+Y)^4-5(X+Y)^2+5\ X^2\ Y^2 \tag{10}$$

to the 7th degree:

$$Z_{227}=(X+Y)^6-7\ X\ Y\ (X+Y)^4+14\ X^2\ Y^2\ (X+Y)^2-7\ X^3\ Y^3 \tag{11}$$

degree n:

$$Z_{22\mathrm{N}}=\frac{X^n+Y^n}{X+Y}=(X+Y)^{n-1}-K_{n-3}XY(X+Y)^{n-3}+...\mp K_2\ X^{\frac{n-3}{2}}\ Y^{\frac{n-3}{2}}\ (X+Y)^2\pm nX^{\frac{n-1}{2}}\ Y^{\frac{n-1}{2}} \tag{12}$$

$$Z_{22\mathrm{N}}=(X+Y)^{n-1}-K_{n-3}XY(X+Y)^{n-3}+...\mp K_2\ X^{\frac{n-3}{2}}\ Y^{\frac{n-3}{2}}(X+Y)^2\pm nX^{\frac{n-1}{2}}\ Y^{\frac{n-1}{2}} \tag{*}$$

Where $K_{n-3}...K_2$ corresponding coefficients at $(XY)^{..}(X+Y)^{..}$ equivalent representation $Z_{22\mathrm{N}}$ algebraic sum of even powers of X+Y and the residual term $\pm n\ X^{\frac{n-1}{2}}\ Y^{\frac{n-1}{2}}$ Note that by n we mean in Mathematical deduction1 any odd power, in Mathematical deduction2 the power of an odd prime number[1-5].

Mathematical deduction1

Suppose that for an n odd number and for the previous n-2 an equivalent representation (*) is valid, then for the next n+2 it is (*) is valid. We show the transition from the two previous odd degrees to the next one.
Further:

$$Z_n^n = X^n + Y^n \tag{13}$$

$$(X^{n-2} + Y^{n-2})(X^2 + Y^2) = X^n + Y^n + Y^2 X^{n-2} + Y^2 Y^{n-2} \tag{14}$$

$$Z_n^n = X^n + Y^n = (X^{n-2} + Y^{n-2})(X^2 + Y^2) - X^2 Y^2 (X^{n-4} + Y^{n-4}) \tag{15}$$

Details:

$\frac{X^{n_1} + Y^{n_1}}{X + Y}$ multiply by $(X + Y)^2 - 2XY$

$$X^{n+2} + Y^{n+2} = (X + Y)[Z_{22N}(X + Y)^2 - 2XYZ_{22N} - X^2 Y^2 (X^{n-2} + Y^{n-2})] \tag{16}$$

$$(X + Y)^{n-1} - K_{n-3} XY (X + Y)^{n-3} + \ldots \mp K_2 X^{\frac{n-3}{2}} Y^{\frac{n-3}{2}} (X + Y)^2 \pm n X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} \tag{**}$$

$$(X + Y)^2 = (X + Y)^{n+1} - K_{n-1(01)} XY (X + Y)^{n-1} + \ldots \mp K_{4(01)} X^{\frac{n-3}{2}} Y^{\frac{n-3}{2}} \pm n X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} (X + Y)^2 \tag{17}$$

$$= -2XY \left[(X + Y)^{n-1} - K_{n-3} XY (X + Y)^{n-3} + \ldots \mp K_2 X^{\frac{n-3}{2}} Y^{\frac{n-3}{2}} (X + Y)^2 \pm n X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} \right]$$

$$= -2XY (X + Y)^{n-1} - K_{n-3(02)} 2 X^2 Y^2 (X + Y)^{n-3} + \ldots \mp K_{2(02)} 2 X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} (X + Y)^2 \pm 2 X^{\frac{n+1}{2}} Y^{\frac{n+1}{2}} n \tag{18}$$

$$-X^2 Y^2 (X + Y)^{n-3} + K_{n-3(03)} X^3 Y^3 (X + Y)^{n-5} + \ldots \pm K_{2(03)} X^{\frac{n-1}{2}} Y^{\frac{n-1}{2}} (X + Y)^2 \mp (n-2) X^{\frac{n+1}{2}} Y^{\frac{n+1}{2}} \tag{19}$$

where

$K_{\ldots(01)}$ - corresponding coefficients when multiplied by $(X+Y)^2$,
$K_{\ldots(02)}$ - corresponding coefficients when multiplied by $-2XY$,
$K_{\ldots(03)}$ -corresponding coefficients when multiplied by $-X^2 Y^2$

After adding these algebraic terms we again obtain(*)
Mathematical deduction 2. The equivalent representation (*) is valid for any prime n.
By Mathematical deduction 1 , if the two previous representations of (*)are valid,of degree 3 and 5, then It is valid for degree 7. Now taking the previous 5 and 7 degrees we have its validity for the 9th degree, etc, which means all odd degrees are described by the above formula and since it includes odd prime, it is valid for prime n.

Let us represent (1) as according to Newton's binomial:

$$(X + Y)^n - Z^n = nX^{n-1} Y + \frac{n(n-1)}{2} X^{n-2} Y^2 + \ldots + \frac{n(n-1)}{2} Y^{n-2} X^2 + n X Y^{n-1}$$

$$= [(X + Y) - Z]^n - n(X + Y) Z * \ldots \tag{20}$$

From (20) it follows that X+Y - Z is divisible by n and further:

$$(X+Y-Z)[(X+Y-Z)^{n-1}-nk_{n-3}(X+Y)Z(X+Y-Z)^{n-3}\pm nk_2(X+Y)^{n-3}Z^{n-3}(X+Y-Z)^2\mp n(XY)^{\frac{n-1}{2}}]$$

$$= nX^{n-1}Y+\frac{n(n-1)}{2}X^{n-2}Y^2+...+\frac{n(n-1)}{2}Y^{n-2}X^2+nXY^{n-1} \quad (21)$$

It follows:

$$(X+Y)[(X+Y)^{n-1}-...\mp n(XY)^{\frac{n-1}{2}}]=Z^n \quad (22)$$

$$Z_{11n}=X+Y \quad (23)$$

$$Z_{22n}=(X+Y)^{n-1}-nk_{n-3}XY(X+Y)^{n-3}+...\pm nk_2X^{\frac{n-3}{2}}Y^{\frac{n-3}{2}}(X+Y)^2\mp nX^{\frac{n-1}{2}}Y^{\frac{n-1}{2}} \quad (24)$$

Analysis of Equation (24)

From equation (24) $Z_{11}=X+Y$ and Z_{22} cannot have a common factor for except for n. Z_{22} consists of members each of which has a factor X+Y, with the exception of the last product $nX^{\frac{n-1}{2}}Y^{\frac{n-1}{2}}$, which in the case of a common factor c
X+Y must involve factors of either X and Y, and they are co-prime, so (25). From which the following equalities follow in the absence of n:

$$X+Y=Z_1^n,\ Z-X=Y_1^n,\ Z-Y=X_1^n \quad (25)$$

$$Z_{11}=Z_1^n,\ Z_{22}=Z_2^n,\ X_{11}=X_1^n,\ X_{22}=X_2^n,\ Y_{11}=Y_1^n,\ Y_{22}=Y_2^n \quad (26)$$

$$X+Y-Z=nX_1Y_1Z_1K_o \quad (27)$$

Where

K_o -an integer coprime to the others specifiedexcept n.

$$Z_1^n=X_1^n+Y_1^n+2nX_1Y_1Z_1K_o \quad (28)$$

$$X-Y=X_1^n-Y_1^n \quad (29)$$

$$Z_1^n-Z=nX_1Y_1Z_1K_o \quad (30)$$

$$Z_2=Z_1^{n-1}-nX_1Y_1K_o \quad (31)$$

$$X-X_1^n=nX_1Y_1Z_1K_o \quad (32)$$

$$X_2=X_1^{n-1}+nZ_1Y_1K_o \quad (33)$$

$$Y - Y_1^n = n\ X_1\ Y_1\ Z_1\ K_o \quad (34)$$

$$Y_2 = Y_1^{n-1} + n\ Z_1\ X_1\ K_o \quad (35)$$

$$2\ X = Z_1^n - Y_1^n + X_1^n \quad (36)$$

$$2\ Y = Z_1^n - X_1^n + Y_1^n \quad (37)$$

$$2\ Z = Z_1^n + X_1^n + Y_1^n \quad (38)$$

$$Z_1^n - X_1^n - Y_1^n = 2\ n\ X_1\ Y_1\ Z_1\ K_o \quad (39)$$

$$Z_1^n - [(X_1 + Y_1)^n - n\ X_1^{n-1}\ Y_1 - \ldots - n\ Y_1^{n-1}\ X_1] = 2\ n\ X_1\ Y_1\ Z_1\ K_o \quad (40)$$

$Z_1 - X_1 - Y_1 = nK_n$ From which it follows $Z_1 > n$ (41)

Note X, Y, Z are co-prime numbers, as well as $X_{1,}\ X_{2,}\ Y_{1,}\ Y_{2,}\ Z_{1,}\ Z_2$

If the sum or difference of two coprime numbers has a factor n, then the sum and difference of the n-power of these numbers is divisible by at least n^2 ,which is obvious from (24), (04).

If in the expansion Z, X, Y has a prime factor n

$$Z_{22} = nZ_2^n\ ,\quad X_{22} = nX_2^n\ ,\quad Y_{22} = nY_2^n \quad (42)$$

and according to formula (24) Z_2 cannot have n available, otherwisethis will lead to the presence of it in X or Y, and vice versa, which is not acceptable.

$Z_2\ , X_2\ , Y_2$ - does not contain the factor n. In this regard, if Z contains a factor n, then formula (30) has the form, since sum $X_1^n + Y_1^n$ contains a multiplier n^m wherenatural number, $m \geq 2$ and $Z_2\ , X_2\ , Y_2$ - does not contain the factor n.

$$n^{nm-1} Z_1^n = X_1^n + Y_1^n + 2\mathrm{n}^m\ X_1\ Y_1\ Z_1\ K_o \quad (43)$$

To solve (38) in integers, degree n in $X_1^n + Y_1^n$,should be equaldegree n in the last monomial, that is, minimally n^2

similar:

$$n^{nm-1}\ X_1^n = Z_1^n - Y_1^n - 2\ n^m\ X_1\ Y_1\ Z_1\ K \quad (44)$$

$$n^{nm-1}\ Y_1^n = Z_1^n - X_1^n - 2\ n^m\ X_1\ Y_1\ Z_1\ K \quad (45)$$

$$n^{nm-1}\ Z_1^n - n^m\ Z_1\ Z_2 = n^m\ X_1\ Y_1\ Z_1\ K\ ,\quad Z = n^m\ Z_1\ Z_2 \quad (46)$$

$$n^m X_1\ X_2 - n^{nm-1}\ X_1^n = n^m\ X_1\ Y_1\ Z_1\ K\ ,\quad X = n^m\ X_1\ X_2 \quad (47)$$

$$n^m Y_1 Y_2 - n^{nm-1} Y_1^n = n^m X_1 Y_1 Z_1 K \; , \; Y = n^m Y_1 Y_2 \tag{48}$$

What follows

$$Z_2 = n^{nm-m-1} Z_1^{n-1} - X_1 Y_1 K \tag{49}$$

$$X_2 = n^{nm-m-1} X_1^{n-1} + Z_1 Y_1 K \tag{50}$$

$$Y_2 = n^{nm-m-1} Y_1^{n-1} + Z_1 X_1 K \tag{51}$$

If there is n in Z, we assign it to some $Z_1 = n^n Z_1^n$

Thus,

$X + Y - Z = n \; X_1 Y_1 Z_1 K_o$ universal,

Where,

$X_{1,} Y_1 \; Z_{1,} K_o$ Co-prime corresponds to X, Y, Z with and without n. The difference is:

$$K_o = n^{m-1} K \tag{52}$$

Since X, Y, Z are relatively prime numbers, the presence of n in one of them obliges the other two to its absence [7].

Degree n=3

According to (32) and Newton's binomial[6]:

$$Z_2^3 = Z_1^6 - 3(X_1^3 + 3 X_1 Y_1 Z_1 K_o)(Y_1^3 + 3 X_1 Y_1 Z_1 K_o) = (Z_1^2 - 3 X_1 Y_1 K_o)^3$$

$$Z_2^3 = Z_1^6 - 9 Z_1^4 X_1 Y_1 K_o + 27 Z_1^2 X_1^2 Y_1^2 K_o^2 - 27 X_1^3 Y_1^3 K_o^3 \tag{53}$$

On the other side :

$$Z_2^3 = (X + Y)^2 - 3 X Y$$

$$= Z_1^6 - 3 X_1^3 Y_1^3 - 9 X_1^3 X_1 Y_1 Z_1 K_o - 9 Y_1^3 X_1 Y_1 Z_1 K_o - 27 X_1^2 Y_1^2 Z_1^2 K_o^2 \tag{54}$$

Underlined in (54) according to (39):

$$-3(X_1^3 + Y_1^3 - Z_1^3 + Z_1^3) 3 X_1 Y_1 Z_1 K_o = 2 * 27 X_1^2 Y_1^2 Z_1^2 K_o^2 - 9 X_1 Y_1 Z_1^4 K_o \tag{55}$$

$$9K_o^3 = 1 \; , \; K_o^3 = \frac{1}{9} \tag{56}$$

There is no solution in whole numbers.

If Z contains n:

$$X + Y = 3^{3m-1} Z_1^3 \tag{57}$$

$$3\,Z_2^3 = 3\left(3^{3m-1}\,Z_1^2 - 3^m\,X_1\,Y_1\,K_o\right)^3 = 3^{9m-2}\,Z_1^6 - 3^{6m+1}\,Z_1^4\,X_1\,Y_1\,K_o +$$
$$+\,3^{5m+1}\,Z_1^2\,X_1^2\,Y_1^2\,K_o^2 - 3^{3m+1}\,X_1^3\,Y_1^3\,K_o^3 \tag{58}$$

$$(X+Y)^2 - 3\,X\,Y = 3^{6m-2}\,Z_1^6 - 3\,X\,Y \tag{59}$$

(58)=(59),when divided by 3^2 there is no solution in integers.

The next solution optionaccording to (24) the difference:

$$(X+Y)^{n-1} - Z_2^n = Z_1^{(n-1)n} - Z_2^n = \left(Z_1^{n-1} - Z_2^n\right)$$

$$\left[\left(Z_1^{n-1} - Z_2\right)^{n-1} - \ldots \pm n Z_1^{(n-1)\left(\frac{n-1}{2}\right)}\,Z_2^{\frac{n-1}{2}}\right] = n^2 \quad \ldots \tag{60}$$

The difference factor (53) n is minimal to the second power. Applicable form=3:

$$Z_1^{2*3} - Z_2^3 = 3\,X_1\,Y_1\,K_o\left(9\,X_1^2\,Y_1^2\,K_o^2 - 3\,X\,Y\right) \tag{61}$$

According to (01):

$$Z_2^3 = (X+Y)^2 - 3\,X\,Y \tag{62}$$

Hence 3XY is divisible by three squared without remainder, and the needanalog X_2 with monomial3ZY or Y_2 3ZX requires that two relatively prime numbers have a factor n, which does not exist. There is no solution in integers.

Degree n.

a) Similar to the third degree, for any degree n, where n is primeodd, the presence of n in one of X, Y, Z will lead to its obligatory presence inthe rest[7]. The confirmation is (24), where in addition to the first polynomial $(X+Y)^{n-1}$ all the rest have a factor of n to the first power, and their sum must contain at least $n^{2.}$ according to (60). The remaining members, except for the last one, contain 2 complete ones from X, Y, Z and one of the elements $X_{11} = Z - Y$, $Y_{11} = Z - X$, $Z_{11} = X + Y$ from the missing third (item 3.b)). Thus, in total they will give at least $n^{2.}$ provided that one of themhas a factor of n, which will lead to the inevitable presence of a factor of n in the other two, which is also impossible, since X,Y,Z are coprime numbers.
b) It is for this case that we examine the balance of the factor n.

Let us consider equation (01) based on (30), (32), (34):

$$\left(X_1^n + n\,X_1\,Y_1\,Z_1\,K_o\right)^n + \left(Y_1^n + n\,X_1\,Y_1\,Z_1\,K_o\right)^n = \left(Z_1^n - n\,X_1\,Y_1\,Z_1\,K_o\right)^n \tag{63}$$

Let's open the brackets:

$$n\,n\,X_1\,Y_1\,Z_1\,K_o\,X_1^{n(n-1)} + n\,n\,X_1\,Y_1\,Z_1\,K_o\,Y_1^{n(n-1)} + n\,n\,X_1\,Y_1\,Z_1\,K_o\,Z_1^{n(n-1)} + \ldots$$
$$\ldots + 2\left(n\,X_1\,Y_1\,Z_1\,K_o\right)^n + X_1^{nn} + Y_1^{nn} - Z_1^{nn} = 0 \tag{64}$$

According to (64), the underlined free term is relatively $n\ X_1\ Y_1\ Z_1\ K_o$ is divisible without remainder by n^{m+1} where m is the degree taking into account its K_o .
Then we have:

$$Z_1^n - X_1^n - Y_1^n \tag{65}$$

It is divisible without remainder by n^m and:

$$Z_1^{nn} - X_1^{nn} - Y_1^{nn} \tag{66}$$

It is divisible without remainder by n^{m+1} .

What should we proceed from (24):

$$Z_1^{nn} - X_1^{nn} - Y_1^{nn} = Z_1^{nn} - (X_1^{nn} + Y_1^{nn}) = Z_1^{nn} - (X_1^n + Y_1^n)^n + n(X_1^n + Y_1^n)^{n-2} - \ldots \pm n(X_1\ Y_1)^{\frac{n-1}{2}}$$

$$(X_1^n + Y_1^n) = Z_1^{nn} - (X_1^n + Y_1^n)^n + n\ X_1^n\ Y_1^n (X_1^n + Y_1^n)^{n-2} - \ldots \pm n(X_1^n\ Y_1^n)^{\frac{n-1}{2}} (X_1^n + Y_1^n) \tag{67}$$

Where the sum of the underlined terms, referring to (24) contains n^{m+1}
Further, according to (28):

$$Z_1^{n^2-1} - (Z_1^{n-1} - 2\ n\ X_1\ Y_1\ K_o)(Z_1^n - 2\ n\ X_1\ Y_1\ Z_1\ K_o)^{n-1} + (Z_1^{n-1} - 2\ n\ X_1\ Y_1\ K_o) * n^{m+1} \ldots \tag{68}$$

and then after opening the brackets (other monomials with a factor n^{m+1}):

$$2\ n\ X_1\ Y_1\ Z_1^{n^2-n}(Z_1^{n-1} - 1) \tag{69}$$

inevitably divides into a simple n. From which it follows according to (31) $Z_2^n - 1$ also divisible without remainder by prime n.

Similarly:

$$X_1[(Z_1^{nn} - Y_1{}^{nn}) - X_1^{nn}] \tag{70}$$

$$\frac{Z_1^n - Y_1^n}{X_1} = \frac{X_1^n + 2\ n\ X_1\ Y_1\ Z_1\ K_o}{X_1} = X_1^{n-1} + 2\ n\ Z_1\ Y_1\ K_o \tag{71}$$

$$(X_1^{n-1} + 2\ Y_1\ Z_1\ K_o)(X_1^n + 2\ n\ X_1\ Y_1\ Z_1\ K_o)^{n-1} - X_1^{n^2-1} \tag{72}$$

$X_1^{n-1} - 1$, $X_2 - X_1^{n-1}$, $X_2 - 1$, $X_2^n - 1$ - are divisible by n (73)

$$(X+Y)^{n-1} - 1 = (X_1^n + Y_1^n + 2\ n\ X_1\ Y_1\ Z_1\ K_o)^{n-1} - 1 = X_1^{(n-1)n} + \ldots + Y_1^{(n-1)n} - 1 \tag{74}$$

Thus, the division of one component is inevitable $X_{1,} Y_{1,} Z_1$ without remainder on n, so that (66) contains n^{m+1} , but there is no solutionpoint 5.a). Otherwise, the free term with respect to p (66) contains a factor n^m and turns into a fraction when reduced in n^{m+1} and there is no solution (01) in integers..

CONCLUSION

If the degree in (01) is odd, there is no solution. Pharm proved the absence of a solution for the 4th degree and thereby proved its absence for everyone $n=2^m$, where m is an integer. Fermat's theorem is solvable in the first and second powers!

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