INTRODUCTION

There are few basics of dynamic programming problems which must be discussed before the details. Those basics are discussed below. A route is defined as a course of travel, especially between two distant points/locations while shortest is sound to be a relatively smallest length, range, scope, etc., than others of its kind, type, etc. Therefore, we say that the shortest route is the relatively smallest of its kind, especially between two distant points/locations. The route must be accessible/useable by a motor vehicle, the route may be single or double lane. The routes may possess bus stops, junctions, interconnected streets, or venues for join. The routes may be traced or not but should be wide enough to be used by a motor vehicle. The routes may short or interconnect with another route that

ABSTRACT

In this research, dynamic programming seeks to address the problem of determining the shortest path between a source and a sink by the method of a fixed point iteration well defined in the metric space \((X,d)\), where \(x = \cup\) the connected series of edges that suitably work with the formula

\[ x_{n+1} = f(x_n) = x^* = F(X) \]

\[ = \text{dist}(S_0, S_k) = \min[U \geq 0, S_0 = \text{source}, S_k = \text{sink}] \]

Such that

\[ d_{ij} \leq d_{ik}, i \neq k, j \neq k, i \neq j \]

with the pivot row and pivot column being row \(k\).

Then, evaluation of the shortest route between Government House and Amuzukwu Primary School all in Umuahia and Abuja by the above method revealed it to be 720 m by taking the route SACDFG.

It was remarked that the longest route which is the route from Government House to Ibiam road, to Aba road, to Warri road, to Club road, to Uwalaka road, and finally to Amuzukwu Road which now terminates at our Destination, Amuzukwu Primary School with Road distance of 2590 m does not possess other advantages while it should be made use of. The shortest routes were necessarily recommended to road users as the best route to use because its route SACDJFGT is the shortest route with the distance of 1790 m.

Key words: Complete metric space, dynamic programming, Dijkstra’s algorithm, Greedy and Prim’s algorithm, pseudo contractive fixed point method, source and node

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INTRODUCTION

There are few basics of dynamic programming problems which must be discussed before the details. Those basics are discussed below. A route is defined as a course of travel, especially between two distant points/locations while shortest is sound to be a relatively smallest length, range, scope, etc., than others of its kind, type, etc. Therefore, we say that the shortest route is the relatively smallest of its kind, especially between two distant points/locations. The route must be accessible/useable by a motor vehicle, the route may be single or double lane. The routes may possess bus stops, junctions, interconnected streets, or venues for join. The routes may be traced or not but should be wide enough to be used by a motor vehicle. The routes may short or interconnect with another route that
started from Umuahia to end at Abuja that is the route should have a source and a sink.[1-3] Therefore, “the application of shortest route/path in Dynamic programming” can be seen as the practical use of the shortest course of travel by road users among other routes of its kind, especially between two points/locations.

It is, however, disturbing to note that much of the available routes from Umuahia to Abuja have one traveling challenge or the other such as long distance, police menace, traffic jams, and bad road networks.[4] Perhaps, it is necessary to answer some questions to really appreciate the issue on ground. These questionnaires are how do road users view the various existing routes from Umuahia to Abuja? Which of the routes are preserved by road users? and What route should one undertake to minimize time and distance of travel? From the researcher’s observation, it was clear that the route quality such as shortest distance freedom from police menace and traffic jam as well as good road networks, all contributed to affect road users decision of route to make use of when traveling from Umuahia to Abuja in Nigeria. Definitely, it is important to note that:

a. This work is limited to time and distance of travel by motor vehicle on road excluding the effects of traffic jams, police menace, bad road network, etc., and number of routes from Umuahia to Abuja.
b. This study will be of major significance to travelers and transporters (who are major beneficiaries).
c. The study will help us appreciate the importance and practical use of dynamic programming in determining the shortest route of travel when traveling from one location to the other.
d. To carry out the study, the following hypothesis was formulated for investigation.

i. Any part of the shortest route from Umuahia to Abuja is itself a shortest path [Table 1].

ii. Any part of an optimal path is itself optimal. The above two hypotheses are also known as “the principle of optimality [Table 2].”

iii. Walk: A walk is simply a route, in the graph along a connected series of edges. BCAD is a walk from B to D through C and ABDE is a walk from A to E in a walk edges and vertices may be repeated [Table 3].

iv. Trail: When all the edges of a walk are different, the walk is called a trail. BCD is a trail from B to D. A closed trail is one in which the start and finish vertices are the same. ADECDBA is a closed trail [Table 4].

v. Path: This is a special kind of trail if all the vertices of a trail are distinct then the trail is a path ABCE is a path, all edges and all vertices are distinct in a path [Table 5].

vi. Cycle: A cycle ends where it starts and all the edges and vertices in between are distinct ABDA and ABCEDA are as vertices have been repeated [Table 6].

vii. Tree: This is a connected graph which contains no cycles. Note that, a tree with n vertices has n – 1 edges.

viii. Vertex Degree: The degree of a vertex is the number of edges touching the vertex

ix. Directed Graph or Diagraph: It is a graph in which each of a diagraph is called an arc

x. Weight: The edges of a graph are often given a number which can represent some physical property, for example, length, cost time, and profit. The general term for this number is weight.

xi. Network: A graph whose edges have all been weighted is called a network.

xii. Stage and State: The stage tells us how “Far” the vertex in question is from the destination vertex while the states refer directly to the vertices.

xiii. Action: This refers to possible choices at each vertex.

xiv. Value: The numbers calculated for each state at each stage are referred to as values.

xv. The Optimal Value: The optimal value is the label which is assigned to the vertex. The value is also known as the Bellman function.

Major introduction [methods of determining the shortest route/path]

There abound several methods of determining the shortest route/path from one location/point to the other in this section we shall do well to review some of the existing methods of finding the shortest route.

The dynamic programming technique

The network below [Robert and Lynda (1999)] can help us explain the dynamic programming technique.
To find the shortest (or longest path from $S$ to $T$ in the above network), we begin at the destination vertex $T$. The vertices next to $T$ best route from these are examined. These are Stage 1 vertex the best route from these to $T$ is noted. We now move to the next set of vertices, moving away from $T$ toward $S$, i.e., the Stage 2 vertices. The optimal route from these vertices to $T$ is found using the already calculated optimal route from the Stage 1 vertices. Then, this process is repeated until the start vertex, $S$, is reached. The optimal route from $S$ to $T$ can then be found that the principle of optimality is used at each stage, the current optimal path is developed from the previously found optimal path.

Since the method involves starting with the destination vertex and working back to start vertex, it is often called backward dynamic programming.

**Dijkstra’s algorithm**

Dijkstra’s algorithm is a method of determining the shortest path between two vertices. The shortest path is found stage by stage. In finding the shortest route to a vertex, we assign to the vertex various numbers. These numbers are simply the length of various paths to that vertex. As there may be many possible paths to a vertex then several different numbers may be assigned to it. Of all possible numbers assigned to a vertex, the smallest one is important. We call this smallest number a label. The label gives the length of the shortest path to the vertex, suppose we wish to find the shortest path from $S$ to $T$ in a network, the algorithm can be presented in three steps. Since the algorithm can be applied to both graphs and digraphs, the word “arc” can be replaced “edge” in the following steps [Taha (2002)]

\[
\begin{align*}
\min [U_j, i] &= \min [U_j, i]; \quad d_B \ge 0,
\end{align*}
\]

**Step 1:** Assign a label $O$ to $S$.

**Step 2:** This is the general step. Look at a vertex which has just been assigned to Label, say the vertex is $A$ via a single arc, say that this vertex is $B$ to $B$ assign the number given by (label of $A = \text{weight} \times AB$). If a vertex is reachable by more than one route assign to it the minimum possible such number. Repeat this process with all vertices that have just been assigned a label and all vertices that are reachable from them. When all reachable vertices have been assigned a number the minimum number is converted into a label. Repeat step 2 until the final vertex $T$ is assigned a label.

**Step 3:** Steps 1 and 2 have simply found the length of the shortest route this step finds the actual shortest route, we begin at the destination vertex $T$ an arc $AB$ is included whenever the condition label $B$ of $A = \text{weight of } AB$ holds true. This route may not be unique.

**Greedy and Prim’s algorithm**

These algorithms are used mainly by television and telephone companies in competing the cities by a cable so that their Carle television and telephone facilities are made available to them, [6] that is, these algorithms help to solve problems known as minimum connector problem, which means connecting cities with minimum amount of cable [Oyeka (1996)]

\[
d_{ij} + d_{ik} < d_{jk}
\]

In graph theory terms, the cities are vertices and the cable is edge. If the vertices are connected in such a way that a cycle exists, then at least one edge could be removed leaving the vertices still connected. Recalling that a connected graph which contains no cycles called a tree, it is clear that the best way of connecting all the vertices would be to find a tree which passes through very vertex. The networks below illustrate this.

A tree which passes through all the vertices of a network is called a spanning tree. Spanning tree which has the shortest total length is a minimum spanning tree. There may be more than one minimum spanning tree. The problem faced by the television or telephone companies is to find a minimum spanning tree of the network.

There are two [Oputa (2005)] algorithms which may be used to find a minimum spanning tree (i) The Greedy algorithm

(ii) Prim’s algorithm

They are essentially the same algorithm and really only differ in the way they are set on. The Greedy algorithm builds up the tree adding one vertex and one edge with each application. Any vertex can be used as a starting the vertex added at each stage unused vertex nearest to any vertex which is already a part of the tree and that the edge added is the shortest available edge. The Greedy algorithm may be summarizing as follows:

**Step 1:** Choose any vertex as a starting vertex.

**Step 2:** Connect the starting vertex to the nearest vertex.
Step 3: Connect the nearest unused vertex to the tree.
Step 4: Repeat step 3 until all vertices have been included.

Prim’s algorithms uses a tabular format making it more suitable for computing purposes since as mentioned earlier greed and Prim’s algorithms are basically the same, it will be enough to illustrate how the Greedy and Prim’s algorithms are used by working through a specific example in chapter three.

BASIC RESULTS

Preliminaries

Let $X$ be a non-empty set and $d$ or $ρ$ a function defined on $X \times X$ into the set of real numbers $R$ such that [Stafford (1969)]

$$d(\ldots) : X \times X \to R$$

satisfying the following conditions

(i) $d(x,y) = 0$ if and only if $x = y$
(ii) $d(x,y) = d(y,x)$ for all $x,y \in X$
(iii) $d(x,y) \leq d(x,z) + d(z,y)$ for all $x, y, z \in X$

The number $d(x,y)$ is called the distance between $x$ and $y$; $d$ is called the metric and the pair $(X,d)$ is called the metric space.

Definition 2.1 [Danbury (1992)]: A subset $A$ of a metric space is said to be bounded if there is a positive constant $M$ such that $d(x,y) \leq M$ for all $x,y \in A$.

Definition 2.2 [Danbury (1992)]: A subset $A$ of a metric space is called a closed set if every convergent sequence in $A$ is limit in $A$.[7]

Definition 2.3 [Chika (2000)]: A subset of a metric space is called compact if every bounded sequence has a convergent subsequence. [8]

Definition 2.4 [Chika (2000)]: A mapping from one metric space into another metric space is called continuous if for every $x_n \to x$ implies that $T(x_n) \to Tx$ that is $\lim d(x_n,x) = 0$ implies $\lim d(T(x_n),Tx) = 0$.

Theorem 2.1 [Robert and Lynda (1999)]: Every bounded and closed subset of $R^n$ is compact.

Definition 2.5 [Stafford (1969)]: A sequence in a metric space $X = (X,d)$ is said to converge or to be convergent if there is an $x \in X$ such that

$$\lim_{n \to \infty} d(x_n,x) = 0 \quad x \text{ is called the limit of } \{x_n\} \text{ and we write } x = \lim_{n \to \infty} x_n$$

or simply $x_n \to x$.

If $\{x_n\}$ is not convergent, it is said to be divergent.

Lemma 2.2 [Chika (2000)]: Let $X = (X,d)$ be a metric space, then

(a) A convergent sequence in $X$ is bounded and its limit is unique
(b) If $x_n \to x$ and $y_n \to y$ in $X$, then $d(x_n,y_n) \to d(x,y)$.

Definition 2.6 [Danbury (1992)]: A sequence $\{x_n\}$ in a metric space $X = (X,d)$ is said to be Cauchy if for every $\varepsilon > 0$, there is an $N = N(\varepsilon)$ such that

$$d(x_n,x_m) < \varepsilon \quad \text{for every } m,n > N$$

The space $X$ is said to be complete if every Cauchy sequence in $X$ converges.

Theorem 2.2 [Chidume (1998)]: The Euclidean space, $R^n$ is a complete metric space.

Definition 2.7 [Chidume (1998)]: A metric can be induced by a norm if a norm on $X$ defines the metric $d$ on $X$ as $d(x,y) = ||x - y||$ and the normed space so defined is denoted by $(X,||.||)$ or simply $X$.

Definition 2.8 [Chika (2000)]: Let $(X,d)$ be a continuous complete metric space with the metric $d(X,X')$ induced by the norm $\|x\|$. If $T : X \to X$ is a map such that

$$Tx = d(x_1,x_2) = x_1 - x_2 = x \quad \forall x_1,x_2 \in X$$

Then, $x$ is a fixed point of the set $X$.

Definition 2.9 [Chika (2000)]: If $x$ be a norm induced by the metric $d$ such that the operator $T : X \to X$ is such that $T_{x_1} - T_{x_2} \leq Kx_1 - x_2 \quad \forall x_1,x_2 \in X$ and $K > 1$, then such a Lipschitzian map is called a contractive map and non-expansive or a pseudocontractive map if, on the other hand, $K = 1$, but if $K > 1$, the map becomes a strong pseudocontraction.

Main Result

The above-mentioned definitions and results served as a guide in developing the facts below which form the basis of our main result used in determining the shortest route problem solutions.

Facts

i) The domain of existence of the shortest route path dynamic programming problem is the complete metric space with the set $X = R$, a closed and bounded set.

ii) The fixed point iterative operator is continuous in the domain of the closed set $R$ and converges at a unique sink ($x_{n+1}$) where the initial iterate $x_0$ is the source [Figures 1-5].
iii) The distance function sometimes is linear and sometimes nonlinear, hence, the reason for the use of the metric induced by the norm $d(x_1,x_2) = |x_1 - x_2|$ [Figure 6].

iv) That the shortest route problem of the dynamic programming problem satisfies the strong pseudo contractive condition of the fixed point iterative method [Figure 7].

v) That the shortest route method of the dynamic programming problem is a reformulation of the modified Krasnoselski’s method of the fixed point iterative method for strongly pseudocontractive maps.

**Theorem 2.3**

Let $(X,d)$ be a complete metric space and $T$ a strongly pseudocontractive iterative map of the shortest route problem in $(X,d)$ induced by the norm $x_1 - x_2$ well posed in the Banach space such that the solution method

$$Tx = \min \left\{ U_j, i \right\} = \sum_j \min \left\{ U_j, i \right\}$$

$$= \sum_j [U_i + d_{ij}, i], d_{ij} \geq 0$$

has the unique fixed point

$$d_{ij} + d_{jk} < d_{ik}$$

With $i \to k$ becoming $i \to j \to k$ and $i \neq k, j \neq k, i \neq j$; the pivot row with pivot column being row $k$ and the triple operation, $i \to j \to k$ holding in each element $d_{ij}$ in $D_{k-1}$ for each $i,j$ such that when $d_{jk} + d_{ij} \leq d_{ij}$ $(i \neq k, j \neq k, i \neq j)$ is satisfied, then we

I. Create $D_k^j$ by replacing $d_{ij}$ in $D_{k-1}$ with $d_{jk} + d_{ij}$

II. Create $S_{k-1}^j$ by replacing $S_{k-1}^j$ in $S_{k-1}$ with $k$ and setting $k$ in $k + 1$ and repeating step $k$.

**Proof**

Let $(X,d)$ be a complete metric space, the closed and bounded distance function space of the dynamic programming containing all the various paths linking the various nodes beginning from the source to the sink [Figure 8-15]. We aim to establish that the dynamic programming method of the shortest route is a strongly pseudocontractive iterative method of the modified Mann. That is, if

$$\|x_1 - x_2\| \leq \left\| (1-r)(x_1 - x_2) - rt(T(x_1) - T(x_2)) \right\|$$

$$= \left\| (1+r)I - rtT \right\| \|x_1 - x_2\| \geq \|x_1 - x_2\|$$

so that

$$\left\| (1+r)I - rtT \right\|$$

and then

$$Tx = \min \left\{ U_j, i \right\} = \sum_j [U_j, i]$$

$$= \sum_j [U_i + d_{ij}, i]; d_{ij} \geq 0 \Rightarrow \sum Kd_{ij} \geq 0$$

Provided $K \geq 0$ where $K$ is the contraction factor. If $K \geq 0$, then the iterative method is strongly pseudocontractive and so the modified Mann’s iterative method in this case the Dijkstra or the Greedy and the Prim’s method becomes the suitable iterative method.

$$x^* = Tx = d_{ij} + d_{jk} < d_{ik}$$

which converges to the unique fixed point whenever $i \Rightarrow k$ is $i \Rightarrow j \Rightarrow k$ and $i \neq k, j \neq k, i = j$; the pivot column becomes row $k$ provided the operation

$$i \Rightarrow j \Rightarrow k$$

holds in each element $d_{ij}$ in $D_{k-1}$ for each $i,j$ such that $d_{jk} + d_{ij} \leq d_{ij}, i \neq k, j \neq k, i = j$ when is satisfied and

i. $D_k^j$ is created by replacing $d_{ij}$ in $D_{k-1}$ with $d_{jk} + d_{ij}$

ii. $S_{k-1}^j$ is created by replacing $s_{ij}$ in $S_{k-1}$ with $k$ and setting $k$ in $k + 1$ and repeating step $k$.

**Applications**

In this section, we should only apply this work to three out of the six reviewed algorithm or methods, i.e.,

(i) Dynamic programming technique

(ii) Dijkstra’s algorithm

(iii) Greedy and Prim’s algorithm

Figure 3 gives the route of study. However, it is important to note that the Government House to Amuzukwu road is a closed and bounded distance network which is continuous in the metric $d(s_g, s_j)$ induced by the norm $\|x - y\|$ such that $x,y \in d(s_g, s_j)$

where $s_g$ is the source. Government house and $s_j$ is the sink, Amuzukwu Primary School, Amuzukwu all in Umuahia. The computation is done using the iterative method of theorem (2.1) above and the sequence of results is displayed in Table 1 and consequently other associated tables that follow [Figure 16-19].
Backward dynamic programming

For ease of reference, we repeat the network drawn in Figure 3 as in Figure 4 be

Where the Dijkstra’s algorithm began at S, the dynamic programming technique work backward from T to S. We begin by considering the vertex joined directly to T, namely G, this is the Stage 1 vertex. The best route from this to T is noted. We now move to the next set of vertices that are joined directly to G, namely F and H – these are Stage 2 vertices. The best route from these to G is found using the optimal routes from the Stage 1 vertices. This process is repeated once again, until S is reached. The principle of optimality is used at each stage and the current optimal path is obtained using the previously obtained optimal paths [Figure 20-25].

Stage 1

From G, there is only one choice and the distance GT is 720 m. We, therefore, label G with 720 m as this is the length of the shortest route to T, also GT is optimal, we indicate it with

\[ \text{Stage 1} \]

\[ \text{From } G, \text{ there are four possible routes } CDEFG, CDJFG, CDEFHG, \text{ and CDJHG} \]

Length CDEFG length of CDEHG + label G = CD+DE+EF+FG+FG+label G = 180+540+150+720 = 2330m

Length CDJFG = length CDJHG + label G = CD+DJ+JH+FG+label G = 180+70+50+600+720 = 1620m

Furthermore, from I to G, we have one route IHG

Length IHG = length IHG + label G = 180+70+50+600+720 = 1620 m

Since we are looking the shortest route, A is assigned the label = min (1760, 1620)

We then have

The stars indicate the optimal routes

Stage 3

From S, there are two choices. We may choose a route through A or I

i) If we choose, A, the shortest route has length = length SA + label A = 170+1620 = 1790 m

ii) If we choose I, the shortest route through has length = length SA + label A = 350+1820 = 2170 m

The shortest route then passes through A and is of length 1790 = min (1790, 2170)

We then have.

The shortest route is obtained by starting at S and using SA, AC, CD, DJ, JF, FG, GT,

\[ S \rightarrow A \rightarrow C \rightarrow D \rightarrow J \rightarrow F \rightarrow G \rightarrow T \]

Application using Dijktra’s algorithm

Furthermore, for reference, we repeat the network drawn in Figure 3 and from their proceed as below.

Step 1

S assigned a label 0, i.e., the shortest distance of S to S is 0 (label is denoted by a number in a box)

A: label S +weight of SA+ O+170 m

I: Label S + weight of SI + O + 350 m

The minimum connector is 170 m; (this is connected into a label) (Numbers are given in brackets).

Step 2

A has just been assigned a label. The vertices reachable from A are B and C and their numbers are calculated.

B: label A +weight of AB = 170+300 = 470 m

C: label A +weight AC = 170+350 = 520 m

We make a label for B

Step 3

B has just been assigned, a label C is reachable from B.

C: label B +weight of BC = 470+ 190 = 660 m

We have also seen
C: label A+ weight of AC = 170+ 350 = 520 m
The minimum number is 520 m and C is assigned
a label of 520.

D label of C+ weight of CD = 520+ 180 = 700 m: So, D is assigned the label of 700 m.

**Step 4**
F: label of D+ weight of DE + weight of
EF = 700+540+ 150 = 1390 m
F: label of D+ weight of J+ weight of F = 700+70+ 90 = 860 m
The minimum number is 860 m, so we can give F label 860

**Step 5**
J: label of D+ weight of DJ = 700+70 = 770 m: J:
label of F+ weight of DJ = 860+90 = 950 m
The minimum number is 770 m: So, we have J labeled 770 m

**Step 6**
I: label of S+ weight of SI = O+350 = 350 m: We
have I labeled 350
H label of I + weight of H = 770+50 = 820 m: We
label H 820 as the minimum number
E and F have been given a label. We now find the number of G: F label G weight G = 860+210 = 1070
G label I + weight IH + weight HG = 770+50+600 = 1420: G label S + weight SI + weight IH = D+350+500+600 = 1450.: The minimum number is 1070, so, we label C.

**Step 7**
G has just given a label. T is only reachable from G and the numbers for T found next. G label of F + weight GT: The minimum number is 1790 M; so, G is given a label 1790 M
At this stage, we know that the shortest distance from S to T is 1790 m. However, we do not yet know the path which achieves his shortest length. Step of algorithm finds that path.

**Step 8**
We start at the destination vertex, T. We include an edge when the weight of the edge is given by the difference of the label of the vertices t the end of the edge.

**Stage 9**
Label of G − label of T = 1790 −1070 include GT = 720M.

**Stage 10**
Label of G − Label of F = 1070 − 860 = 210 m:
Weight of FG = 210 include FG
Label of G − Label of I = 1070 − 35 = 72: Weight
of IG = 1100) do not include IG
Label of G − Label H + weight of HG = 1070
-(820+600) = 1070 − 1420 = −350) do not include JG

**Stage 11**
Label of D − Label of C = 860−770 = 90
(Weight of JF = 90) include JF
Label of G − Label of H = 860 − 124
Weight of HG = 150) do not include IJ

**Stage 12**
Label J − Label of D = 770 − 700 = 70
(Weight of DJ = 70) include DJ

**Stage 13**
Label D − Label of C = 700 − 520 = 180
WeightCD = 180) include CD

**Stage 14**
Label C − Label B + weight of BC 520 − (470 + 190) 529 − 660 = −140
Weight CA = 490) Do not include CA
Label C − label A 520 − 170 = 350: (Weight of CA = 350) include CA.

**Stage 15**
Label A − labels = 170 − 0 = 170
(Weight of SA = 170) include SA (Label IS = 350)
do not include IS. Hence, the shortest route from S to T is SACDJFGT.

**Application using Greedy and Prim’s algorithm**

*Application by Greedy’s algorithm*

Applying Greedy’s algorithm in the figure, i.e., Figure 3, we have been considering. We use the following procedure.

Choosing any vertex as a starting vertex, say S
The nearest vertex to S is A
The nearest vertex to A is C
Also the nearest vertex to A is C
The nearest vertex to D is J
The nearest vertex to J is F
The nearest vertex to F is G
The nearest and only vertex to G is T
The total length of the figure is 4470 m, in this example, the minimum spanning tree is not unique
since at each step, in algorithm, we have an alternative in deciding the next vertex and edge. The shortest route is the path SACDJFGT with 1790 metres distance.

**Application using Prim’s algorithm**
Prim’s Algorithm uses the table format below to find the shortest route (Table 1).
Reorganize the table to take vertical shape and herisenta. From a close study of the table above, we come up with the following resolutions,

i. There is a zero distance from S to S; therefore, we eliminate row and column S

ii. The nearest distance from A to A so we eliminate row and column A

iii. We resolve to make use of points ACDEFGT, ACDJEGT, and ACDJHGT, hereby over looking point S in our further steps for ease of possible manipulation of the data and table to give accurate result.

iv. We represent the shortest route from one point to another with the least number derivable in that route.

We now illustrate, the shortest route chosen as follows:
A critical look into the table above we’ll help us resolve as follows:

i. That AC and CD remains constant with value 350 and 180, then we eliminate A and C

ii. The shortest route diagram includes SACD
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The smallest number in the F and G row is 210, respectively. Hence, we include G in the shortest route diagram as thus SACDJFG we eliminate F. Then, table becomes or reduces to the smallest and only number in the G row is 720, representing T, so, we include T in the shortest route diagram and elimination. The shortest route diagram now becomes SACDJFG with shortest route distance of 1790 m. The shortest route diagram is now represented by

**Remark**

With all what have done in the three algorithms or methods of shortest route, we took time to apply, we can see clearly that three methods got a particularly shortest route proving the accuracy of the work and showing the methods were rightly applied [Figure 26-18]. Hence, we want to say here that irrespectively of the algorithm or method you may want to use, the shortest should remain the same except for methods like the Eulerian and non-Eulerian graph, min-max and max-min route were some additions would be made on the shortest route, but irrespective of the additions, the fundamental shortest route will remain the same [Figure 29-31].

Note: By inspection, the longest route is the route from government house through Ibiam road to Aba road then to Warri road also through Club road to Uwalaka road and finally to Amuzukwu road which terminate at destination, Amuzukwu Primary School Umuahia. The longest route covered a total distance of 2590 metres which passed through the path SIHJDEFT, i.e., [Figure 32-35]

\[ S \rightarrow I \rightarrow H \rightarrow J \rightarrow D \rightarrow E \rightarrow F \rightarrow G \rightarrow T \]

**DISCUSSION, CONCLUSION, AND SUGGESTIONS**

The minimum length of travel on any route goes a long way in determining the route of travel of road user where the case of alternative routes of travel exists. It is true that there exists other factors competing with minimum length in winning the choice of the route to use such as security, good road network, absence of police menace, and traffic jams, still in a city such as Umuahia where every thing is done to save time and for the purpose of this study where other factors were put on constraints, we have no other alternative but to appreciate
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The good gesture done to us by the application of shortest path to dynamic programming. The application of shortest route in dynamic programming has been attributed to some factors which we have listed in the course of this work. This chapter, therefore, discussed and summaries the findings of the study and makes conclusion based on empirical findings of how shortest route is applied in dynamic programming. Hypothesis was put forward and analyzed using mathematical tool of application which explained the data collected in the course of the study.
Discussion of finding

Our finding on choice of routes, road user within Umuahia metropolis makes use of showed that most Umuahia road users have one problem or the other traveling on road. The problems range from potholes, traffic jams, and long distance route. Existing route is even been covered by market and traders, thereby increasing traffic jam. Our finding also discovered that road users and motor vehicles are increasing at geometric progression which the route/networks are increasing
at arithmetic progression or even estimated not increasing. Flood, during rainy season, contributed its quota to hinder minimum length and time travel. Thus, putting other factors affecting movement from one point to the other in constraint and focusing on distance, we will certainly agree that shortest route makes travel interesting. These follow the hypothesis that any path of shortest route is itself a shortest path and we say that any part of an optimal route is itself optimal.

**Suggestion**

Based on our findings in the course of this study, the researcher suggests as follows:

1. That route is created from one geographical location to another by any responsible authority, especially government.

2. Maintenance activity/works should be done on a regular basis on the existing routes.

3. Road directions and warnings should be positioned at strategic junctions to enable travelers locate their destination from their source and have enough information to prevent accidents.

4. Branched network should be attached to reduce the rate of traffic jams on our route.

5. Police menace on our road (routes) of travel should be discouraged.

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6. Road users should be cautious as they the road.
7. Safety providing agencies should make themselves available in every route of travel within Umuahia.
8. Road users should make the shortest path their route of travel to minimize length and time of travel.

CONCLUSION

In the transportation world today, the routes are regarded as king in the sense that they provide channels/links between two geographical points/location. The routes do not just come into existence. They are created or built by men to facilitate movement from one point to the other. Though these routes cannot catapult any one geographical location to the other on their own by when they exist, and good once, even without locomotion machines like motor vehicles one can still make a journey by foot. Government on their own should make building and maintenance of roads and networks paramount projects. It is expected that Wise Travelers having known that there exists short and long route may decide to choose traveling through the shortest route.

Suggestions for further research

This work has examined the application of shortest route in dynamic programming considering the factors of minimum distance. Further studies could still be carried out to understand more factors which could likely determine the minimum or maximum distance between two locations and other applications excluding the one used in this work to determine the shortest path/route between one location/point to the other.

REFERENCES