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# On Decreasing of Dimensions of Field-Effect Heterotransistors in Logical CMOP Voltage Differencing Inverting Buffered Amplifier Manufactured

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## ABSTRACT

In this paper, we introduce an approach to decrease the dimensions of CMOP voltage differencing inverting buffered amplifier based on field-effect heterotransistors by increasing density of elements. Dimensions of the elements will be decreased due to manufacture heterostructure with a specific structure, doping of required areas of the heterostructure by diffusion or ion implantation, and optimization of annealing of dopant and/or radiation defects.

**Key words:** CMOP voltage differencing inverting buffered amplifier, increasing integration rate of field-effect heterotransistors, optimization of manufacturing

## INTRODUCTION

At present, density of the elements of integrated circuits and their performance intensively increases. Simultaneously, with increasing of the density of the elements of an integrated circuit, their dimensions decrease. One way to decrease dimensions of these elements of these integrated circuits is manufacturing of these elements in thin-film heterostructures.[1-4] An alternative approach to decrease the dimensions of the elements of integrated circuits is using laser and microwave types annealing.[5-7] Using these types of annealing lead to generation inhomogeneous distribution of temperature. Due to Arrhenius law the inhomogeneity of the diffusion coefficient and other parameters of process dimensions of elements of integrated circuits decreases. The inhomogeneity gives us possibility to decrease dimensions of the elements of integrated circuits. Changing of the properties of electronic materials could be obtained using radiation processing of these materials.[8,9]

In this paper, we consider CMOP voltage differencing inverting buffered amplifier based on field-effect transistors described in Pushkar [Figure 1].[10] We assume that the considered element has been manufactured in heterostructure from Figure 1. The heterostructure consists of a substrate and an epitaxial layer. The epitaxial layer includes into itself several sections manufactured using another material. The sections have been doped by diffusion or ion implantation to generation into these sections required type of conductivity ($n$ or $p$). Framework this paper, we analyzed redistribution of dopant during annealing of dopant and/or radiation defects to formulate conditions for decreasing of dimensions of the considered amplifier.

### Method of solution

We determine spatiotemporal distribution of the concentration of dopant by solving the following boundary problem.

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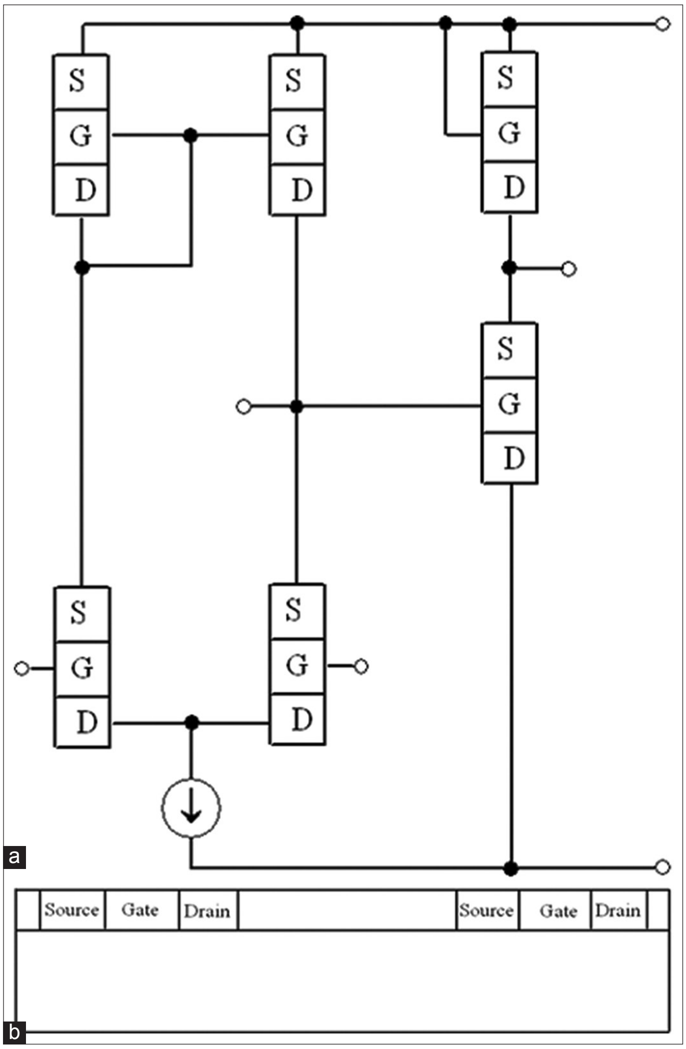


**Figure 1:** (a) Structure of considered amplifier. View from top. (b) Heterostructure with two layers and sections in the epitaxial layer

$$\frac{\partial\, C(x,y,z,t)}{\partial\, t}=$$

$$=\frac{\partial}{\partial\, x}\left[D_C\frac{\partial\, C(x,y,z,t)}{\partial\, x}\right]+\frac{\partial}{\partial\, y}\left[D_C\frac{\partial\, C(x,y,z,t)}{\partial\, y}\right]+\frac{\partial}{\partial\, z}\left[D_C\frac{\partial\, C(x,y,z,t)}{\partial\, z}\right] \tag{1}$$

with boundary and initial conditions

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0}=0, \tag{2}$$

$$\left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{x=L_y}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{x=L_z}=0,\ C\,(x,y,z,0)=f\,(x,y,z).$$

Here, $C(x,y,z,t)$ is the spatiotemporal distribution of the concentration of dopant; $T$ is the temperature of annealing; $D_C$ is the dopant diffusion coefficient. Value of dopant diffusion coefficient depends on properties of materials and speed of heating and cooling of heterostructure (with account Arrhenius law). Dependences of dopant diffusion coefficients could be approximated by the following function.[9,11,12]

$$D_C = D_L\left(x,y,z,T\right)\left[1+\xi\frac{C^{\gamma}\left(x,y,z,t\right)}{P^{\gamma}\left(x,y,z,T\right)}\right]\left[1+\varsigma_1\frac{V\left(x,y,z,t\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,t\right)}{\left(V^*\right)^2}\right], \tag{3}$$

where $D_L(x,y,z,T)$ is the spatial (due to existing several layers with different properties in heterostructure) and temperature (due to Arrhenius law) dependences of dopant diffusion coefficient; $P(x,y,z,T)$ is the limit of solubility of dopant; parameter $\gamma$ could be integer framework the following interval $\gamma\in$[1,3,9]; $V(x,y,z,t)$ is the spatiotemporal distribution of the concentration of radiation vacancies; $V^*$ is the equilibrium distribution of the concentration of vacancies. Concentration dependence of dopant diffusion coefficient has been discussed in details in Vinetskiy.[9] It should be noted that using diffusion type of doping did not lead to generation radiation defects and $\zeta_1=\zeta_2=0$. We determine that spatiotemporal distributions of the concentrations of point defects have been determined by solving the following system of equations.[11,12]

$$\frac{\partial I\left(x,y,z,t\right)}{\partial t}=\frac{\partial}{\partial x}\left[D_I\left(x,y,z,T\right)\frac{\partial I\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I\left(x,y,z,T\right)\frac{\partial I\left(x,y,z,t\right)}{\partial y}\right]$$

$$+\frac{\partial}{\partial z}\left[D_I\left(x,y,z,T\right)\frac{\partial I\left(x,y,z,t\right)}{\partial z}\right]-k_{I,V}\left(x,y,z,T\right)I\left(x,y,z,t\right)V\left(x,y,z,t\right)$$

$$-k_{I,I}\left(x,y,z,T\right)I^2\left(x,y,z,t\right) \tag{4}$$

$$\frac{\partial V\left(x,y,z,t\right)}{\partial t}=\frac{\partial}{\partial x}\left[D_V\left(x,y,z,T\right)\frac{\partial V\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V\left(x,y,z,T\right)\frac{\partial V\left(x,y,z,t\right)}{\partial y}\right]$$

$$+\frac{\partial}{\partial z}\left[D_V\left(x,y,z,T\right)\frac{\partial V\left(x,y,z,t\right)}{\partial z}\right]-k_{I,V}\left(x,y,z,T\right)I\left(x,y,z,t\right)V\left(x,y,z,t\right)$$

$$-k_{V,V}\left(x,y,z,T\right)V^2\left(x,y,z,t\right)$$

with boundary and initial conditions

$$\left.\frac{\partial\rho\left(x,y,z,t\right)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial\rho\left(x,y,z,t\right)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial\rho\left(x,y,z,t\right)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial\rho\left(x,y,z,t\right)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial\rho\left(x,y,z,t\right)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial\rho\left(x,y,z,t\right)}{\partial z}\right|_{z=L_z}=0,\ \rho\left(x,y,z,0\right)=f_{\rho}\left(x,y,z\right). \tag{5}$$

Here, $\rho=I,V$; $I(x,y,z,t)$ is the spatiotemporal distribution of the concentration of radiation interstitials; $D_{\rho}(x,y,z,T)$ is the diffusion coefficients of radiation interstitials and vacancies; terms $V^2(x,y,z,t)$ and $I^2(x,y,z,t)$ correspond to generation of divacancies and di-interstitials; $k_{I,V}(x,y,z,T)$ is the parameter of recombination of point radiation defects; $k_{\rho,\rho}(x,y,z,T)$ is the parameter of generation of simplest complexes of point radiation defects.

We determine spatiotemporal distributions of the concentrations of divacancies $\Phi_V(x,y,z,t)$ and di-interstitials $\Phi_I(x,y,z,t)$ by solving the following system of equations.[11,12]

$$\frac{\partial\,\Phi_I\left(x,y,z,t\right)}{\partial\,t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}\left(x,y,z,T\right)\frac{\partial\Phi_I\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}\left(x,y,z,T\right)\frac{\partial\Phi_I\left(x,y,z,t\right)}{\partial y}\right]$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)-k_I(x,y,z,T)I(x,y,z,t) \tag{6}$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)-k_V(x,y,z,T)V(x,y,z,t)$$

with boundary and initial conditions

$$\left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial z}\right|_{z=L_z}=0, \tag{7}$$

$\Phi_I$ (x,y,z,$0$)=$f_{\Phi I}$(x,y,z), $\Phi_V$ (x,y,z,$0$)=$f_{\Phi V}$(x,y,z).

Here, $D_{\Phi\rho}(x,y,z,T)$ is the diffusion coefficients of complexes of radiation defects; $k_\rho(x,y,z,T)$ is the parameters of decay of complexes of radiation defects.

We determine spatiotemporal distributions of the concentrations of dopant and radiation defects using the method of averaging of function corrections[13] with decreased quantity of iteration steps.[14] Framework the approach, we used solutions of Equations (1), (4), and (6) in linear form and with averaged values of diffusion coefficients $D_{0L}$, $D_{0I}$, $D_{0V}$, $D_{0\Phi I}$, and $D_{0\Phi V}$ as initial order approximations of the required concentrations. The solutions could be written as

$$C_1(x,y,z,t)=\frac{F_{0C}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(t),$$

$$I_1(x,y,z,t)=\frac{F_{0I}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{nI}c_n(x)c_n(y)c_n(z)e_{nI}(t),$$

$$V_1(x,y,z,t)=\frac{F_{0C}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{nC}c_n(x)c_n(y)c_n(z)e_{nV}(t),$$

$$\Phi_{V1}(x,y,z,t)=\frac{F_{0\Phi_V}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{n\Phi_V}c_n(x)c_n(y)c_n(z)e_{n\Phi_V}(t),$$

where $e_{n\rho}(t)=\exp\left[-\pi^2n^2D_{0\rho}t\left(\frac{1}{L_x^2}+\frac{1}{L_y^2}+\frac{1}{L_z^2}\right)\right]$, $F_{n\rho}=\int\limits_0^{L_x}c_n(u)\int\limits_0^{L_y}c_n(v)\int\limits_0^{L_z}c_n(v)f_\rho(u,v,w)\,d\,w\,d\,v\,d\,u,$

$c_n(\chi) = cos\ (\pi\ n\ \chi/L_\chi)$.

The second-order approximations and approximations with higher orders of the concentrations of dopant and radiation defects, we determine framework standard iterative procedure.[13,14] Framework this procedure to calculate approximations with the $n$-order, one should replace the functions $C(x,y,z,t)$, $I(x,y,z,t)$, $V(x,y,z,t)$, $\Phi_I(x,y,z,t)$, $\Phi_V(x,y,z,t)$ in the right sides of the Equations (1), (4), and (6) on the following sums $\alpha_{n\rho}+\rho_{n-1}(x,y,z,t)$. As an example, we present equations for the second-order approximations of the considered concentrations.

$$\frac{\partial C_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left(\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right.$$
$$\left.\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_L(x,y,z,T)\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\right.$$
$$\left.\times\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\frac{\partial C_1(x,y,z,t)}{\partial y}\right)+\frac{\partial}{\partial z}\left(D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial z}\right.$$
$$\left.\times\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right) \tag{8}$$

$$\begin{cases}\dfrac{\partial I_2(x,y,z,t)}{\partial t}=\dfrac{\partial}{\partial x}\left[D_I(x,y,z,T)\dfrac{\partial I_1(x,y,z,t)}{\partial x}\right]+\dfrac{\partial}{\partial y}\left[D_I(x,y,z,T)\dfrac{\partial I_1(x,y,z,t)}{\partial y}\right]\\ +\dfrac{\partial}{\partial z}\left[D_I(x,y,z,T)\dfrac{\partial I_1(x,y,z,t)}{\partial z}\right]-[\alpha_{2I}+I_1(x,y,z,t)][\alpha_{2V}+V_1(x,y,z,t)]\\ \times k_{I,V}(x,y,z,T)-k_{I,I}(x,y,z,T)[\alpha_{2I}+I_1(x,y,z,t)]^2\\ \dfrac{\partial V_2(x,y,z,t)}{\partial t}=\dfrac{\partial}{\partial x}\left[D_V(x,y,z,T)\dfrac{\partial V_1(x,y,z,t)}{\partial x}\right]+\dfrac{\partial}{\partial y}\left[D_V(x,y,z,T)\dfrac{\partial V_1(x,y,z,t)}{\partial y}\right]\\ +\dfrac{\partial}{\partial z}\left[D_V(x,y,z,T)\dfrac{\partial V_1(x,y,z,t)}{\partial z}\right]-[\alpha_{2I}+I_1(x,y,z,t)][\alpha_{2V}+V_1(x,y,z,t)]\\ \times k_{I,V}(x,y,z,T)-k_{V,V}(x,y,z,T)[\alpha_{2V}+V_1(x,y,z,t)]^2\end{cases} \tag{9}$$

$$\begin{cases}\dfrac{\partial \Phi_{I2}(x,y,z,t)}{\partial t}=\dfrac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\dfrac{\partial \Phi_{I1}(x,y,z,t)}{\partial x}\right]+\dfrac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\right.\\ \left.\times\dfrac{\partial \Phi_{I1}(x,y,z,t)}{\partial y}\right]+\dfrac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\dfrac{\partial \Phi_{I1}(x,y,z,t)}{\partial z}\right]+k_{I,I}(x,y,z,T)\\ \times I^2(x,y,z,t)-k_I(x,y,z,T)I(x,y,z,t)\\ \dfrac{\partial \Phi_{V2}(x,y,z,t)}{\partial t}=\dfrac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\dfrac{\partial \Phi_{V1}(x,y,z,t)}{\partial x}\right]+\dfrac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\right.\\ \left.\times\dfrac{\partial \Phi_{I1}(x,y,z,t)}{\partial y}\right]+\dfrac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\dfrac{\partial \Phi_{V1}(x,y,z,t)}{\partial z}\right]+k_{V,V}(x,y,z,T)\\ \times V^2(x,y,z,t)-k_V(x,y,z,T)V(x,y,z,t)\end{cases} \tag{10}$$

Integration of the left and right sides of Equations (8)–(10) gives us possibility to obtain relations for the second-order approximations of the concentrations of dopant and radiation defects in final form.

$$C_2(x,y,z,t)=\frac{\partial}{\partial x}\left(\int_0^t\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right.$$

$$\left.\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,\tau)}{\partial x}\right)d\tau+\frac{\partial}{\partial y}\left(\int_0^t\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\right.$$

$$\left.\times D_L(x,y,z,T)\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\frac{\partial C_1(x,y,z,\tau)}{\partial y}d\tau\right)+\frac{\partial}{\partial z}\left(\int_0^t D_L(x,y,z,T)\right.$$

$$\left.\times\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\frac{\partial C_1(x,y,z,\tau)}{\partial z}d\tau\right)$$

$$+f_C(x,y,z) \tag{8a}$$

$$I_2(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_I(x,y,z,T)\right.$$

$$\left.\times\frac{\partial I_1(x,y,z,\tau)}{\partial y}d\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial z}d\tau\right]-\int_0^t k_{I,I}(x,y,z,T)$$

$$\times[\alpha_{2I}+I_1(x,y,z,\tau)]^2d\tau+f_I(x,y,z)-\int_0^t k_{I,V}(x,y,z,T)[\alpha_{2I}+I_1(x,y,z,\tau)]$$

$$\times[\alpha_{2V}+V_1(x,y,z,\tau)]d\tau \tag{9a}$$

$$V_2(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_V(x,y,z,T)\right.$$

$$\left.\times\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial z}d\tau\right]-\int_0^t k_{V,V}(x,y,z,T)$$

$$\times[\alpha_{2I}+V_1(x,y,z,\tau)]^2d\tau+f_V(x,y,z)-\int_0^t k_{I,V}(x,y,z,T)[\alpha_{2I}+I_1(x,y,z,\tau)]$$

$$\times[\alpha_{2V}+V_1(x,y,z,\tau)]d\tau$$

$$\Phi_{I2}(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{I1}(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_{\Phi_I}(x,y,z,T)\right.$$

$$\left.\times\frac{\partial \Phi_{I1}(x,y,z,\tau)}{\partial y}d\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{I1}(x,y,z,\tau)}{\partial z}d\tau\right]+\int_0^t k_{I,I}(x,y,z,T)$$

$$\times I^2(x,y,z,\tau)d\tau-\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)d\tau+f_{\Phi_I}(x,y,z) \quad (10a)$$

$$\Phi_{V2}(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{V1}(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_{\Phi_V}(x,y,z,T)\right.$$

$$\left.\times\frac{\partial \Phi_{I1}(x,y,z,\tau)}{\partial y}d\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{V1}(x,y,z,\tau)}{\partial z}d\tau\right]+\int_0^t k_{V,V}(x,y,z,T)$$

$$\times V^2(x,y,z,\tau)d\tau-\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)d\tau+f_{\Phi_V}(x,y,z)$$

We determine average values of the second-order approximations of the considered concentrations using the following standard relations.[13,14]

$$\alpha_{2\rho}=\frac{1}{\Theta L_x L_y L_z}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[\rho_2(x,y,z,t)-\rho_1(x,y,z,t)\right]d\,z\,d\,y\,d\,x\,d\,t. \quad (11)$$

Substitution of relations (8*a*)–(10*a*) into relation (11) gives us possibility to obtain relations for the required average values $\alpha_{2\rho}$

$$\alpha_{2C}=\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_C(x,y,z)\,d\,z\,d\,y\,d\,x, \quad (12)$$

$$\alpha_{2I}=\frac{1}{2A_{II00}}\left\{\left(1+A_{IV01}+A_{II10}+\alpha_{2V}A_{IV00}\right)^2-4A_{II00}\left[\alpha_{2V}A_{IV10}-A_{II20}+A_{IV11}\right.\right.$$

$$\left.\left.-\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,d\,z\,d\,y\,d\,x\right]\right\}^{\frac{1}{2}}-\frac{1+A_{IV01}+A_{II10}+\alpha_{2V}A_{IV00}}{2A_{II00}} \quad (13a)$$

$$\alpha_{2V}=\frac{1}{2B_4}\sqrt{\frac{(B_3+A)^2}{4}-4B_4\left(y+\frac{B_3y-B_1}{A}\right)}-\frac{B_3+A}{4B_4}, \quad (13b)$$

where $A_{abij}=\frac{1}{\Theta L_x L_y L_z}\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} k_{a,b}(x,y,z,T)\,I_1^i(x,y,z,t)\,V_1^j(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t,$

$$B_4 = A_{IV00}^2 A_{IV00}^2 - 2\left(A_{IV00}^2 - A_{II00}A_{VV00}\right)^2,\ B_3 = A_{IV00}A_{IV00}^2 + A_{IV01}A_{IV00}^3 + A_{IV00}A_{II10}A_{IV00}^2$$

$$-4\left(A_{IV00}^2 - A_{II00}A_{VV00}\right)\left[2A_{IV01}A_{IV00} + 2A_{IV00}\left(1 + A_{IV01} + A_{II10}\right) - 2A_{II00}\left(A_{IV10} + A_{VV10} + 1\right)\right]$$

$$-4A_{IV10}A_{II00}A_{IV00}^2 + 2A_{IV00}A_{IV01}A_{IV00}^2,\ B_2 = A_{IV00}^2\left\{\left(1 + A_{IV01} + A_{II10}\right)^2 + A_{IV00}^2 A_{IV01}^2 + A_{IV00}\right.$$

$$\times 2A_{IV00}\left(A_{IV00} + A_{IV00}A_{IV01} + A_{IV00}A_{II10} - 4A_{IV10}A_{II00}\right) - 4A_{II00}\left[A_{IV11} - A_{II20} - \frac{1}{L_xL_yL_z}\right.$$

$$\left.\left.\times\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx\right]\right\}\left\{\left[2A_{IV01}A_{IV00} + 2A_{IV00}\left(1 + A_{IV01} + A_{II10}\right) - 2A_{II00}\left(A_{IV10} + 1\right.\right.\right.$$

$$\left.\left.+A_{VV10}\right)\right]^2 + 2\left[A_{IV01}\left(1 + A_{IV01} + A_{II10}\right) + \frac{2}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_V(x,y,z)\,dz\,dy\,dx - 2A_{II00}\left(A_{VV20}\right.\right.$$

$$\left.\left.-A_{IV11}\right) + A_{IV01}\left(1 + A_{IV01} + A_{II10}\right)\right]\left[2A_{IV00}\left(1 + A_{IV01} + A_{II10}\right) + 2A_{IV01}A_{IV00} - 2A_{II00}\left(A_{IV10} + 1 +\right.\right.$$

$$\left.\left.\left.+A_{VV10}\right)\right]\right\},\ B_1 = 2A_{IV00}A_{IV01}\left(1 + A_{IV01} + A_{II10}\right)^2 - 8\left[A_{IV11} - \frac{1}{L_x}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx\right.$$

$$\left.\times\frac{1}{L_yL_z} - A_{II20}\right] + A_{IV00}A_{IV01}A_{II00} + A_{IV01}^2\left(A_{IV00} + A_{IV00}A_{IV01} + A_{IV00}A_{II10} - 4A_{IV10}A_{II00}\right)$$

$$-2\left[\frac{2A_{II00}}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx + A_{IV01}\left(1 + A_{IV01} + A_{II10}\right) - 2A_{II00}\left(A_{VV20} - A_{IV11}\right)\right.$$

$$\left.+A_{IV01}\left(1 + A_{IV01} + A_{II10}\right)\right]\left[2A_{IV00}\left(1 + A_{IV01} + A_{II10}\right) - 2\left(A_{IV10} + A_{VV10} + 1\right)A_{II00} + 2A_{IV01}A_{IV00}\right],$$

$$B_0 = 4A_{II00}A_{IV01}^2\left[A_{II20} + \frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx - A_{IV11}\right] + A_{IV01}^2\left(A_{IV01} + A_{II10}\right.$$

$$\left.+1\right)^2 - \left[\frac{2A_{II00}}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_V(x,y,z)\,dz\,dy\,dx + A_{IV01}\left(1 + A_{IV01} + A_{II10}\right) - 2A_{II00}\left(A_{VV20} - A_{IV11}\right)\right.$$

$$\left.+A_{IV01}\left(1 + A_{IV01} + A_{II10}\right)\right]^2,\ y = \sqrt[3]{\sqrt{q^2 + p^3} - q} - \sqrt[3]{\sqrt{q^2 + p^3} + q} + \frac{B_2}{6},\ q = \left(2B_1B_3 - 8B_0\right)$$

$$\times\frac{B_2}{48} + \frac{B_2^3}{216} + \frac{B_0\left(4B_2 - B_3^2\right) - B_1^2}{8},\ p = \left[3\left(2B_1B_3 - 8B_0\right) - 2B_2^2\right]/72,\ A = \sqrt{8y + B_3^2 - 4B_2},$$

$$\alpha_{2\Phi_I} = A_{II20} - \frac{1}{\Theta L_x L_y L_z}\int_0^{\Theta}(\Theta - t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} k_I(x,y,z,T)\,I(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t +$$

$$+\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_{\Phi I}(x,y,z)\,d\,z\,d\,y\,d\,x \quad (14)$$

$$\alpha_{2\Phi_V} = A_{VV20} - \frac{1}{\Theta L_x L_y L_z}\int_0^{\Theta}(\Theta - t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} k_V(x,y,z,T)\,V(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t$$

$$+\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_{\Phi V}(x,y,z)\,d\,z\,d\,y\,d\,x.$$

The considered substitution gives us possibility to obtain the equation for parameter $a_{2C}$. Solution of the equation depends on value of parameter $\gamma$. Analysis of spatiotemporal distributions of concentrations of dopant and radiation defects has been done using their second-order approximations framework the method of averaged of function corrections with decreased quantity of iterative steps. The second-order approximation is usually enough good approximation to make qualitative analysis and obtain some quantitative results. Results of analytical calculation have been checked by comparison with results of numerical simulation.

## DISCUSSION

In this section, we analyzed the spatiotemporal distribution of the concentration of dopant in the considered heterostructure during annealing. Figure 2 shows spatial distributions of concentrations of dopants infused [Figure 2a] or implanted [Figure 2b] in epitaxial layer. Value of annealing time is equal for all distributions framework every Figure 2a and b. Increasing of the number of curves corresponds to increasing of difference between values of dopant diffusion coefficients in layers of heterostructure.

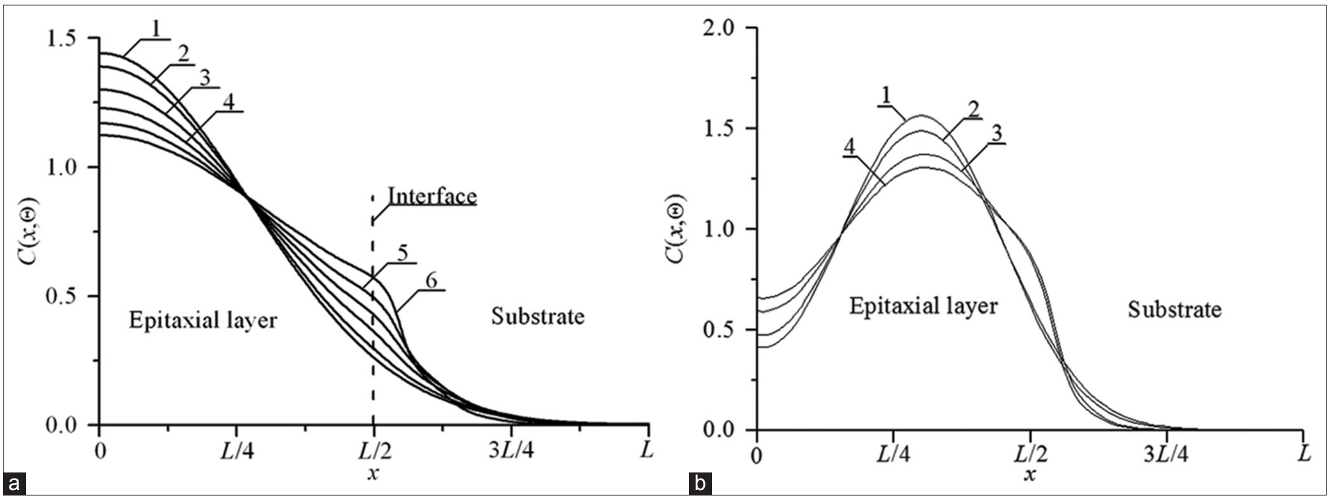


**Figure 2:** (a) Distributions of the concentration of infused dopant in heterostructure from Figures 1 and 2 in direction, which is perpendicular to interface between epitaxial layer substrate. Increasing of the number of curve corresponds to increasing of difference between values of dopant diffusion coefficient in layers of heterostructure under condition when the value of dopant diffusion coefficient in epitaxial layer is larger the than value of dopant diffusion coefficient in substrate. (b) Distributions of the concentration of implanted dopant in heterostructure from Figures 1 and 2 in direction, which is perpendicular to interface between epitaxial layer substrate. Curves 1 and 3 correspond to annealing time $\Theta = 0.0048(L_x^2+L_y^2+L_z^2)/D_0$. Curves 2 and 4 correspond to annealing time Q = $0.0057(L_x^2+L_y^2+L_z^2)/D_0$. Curves 1 and 2 correspond to homogeneous sample. Curves 3 and 4 correspond to heterostructure under condition when the value of dopant diffusion coefficient in epitaxial layer is larger than the value of dopant diffusion coefficient in substrate

The figures show that the presence of interface between layers of heterostructure gives us possibility to increase absolute value of gradient of the concentration of dopant in direction, which is perpendicular to the interface. We obtain increasing of absolute value of the gradient in neighborhood of the interface. Due to the increasing, one can obtain decreasing dimensions of transistors, which have been used in the amplifier. At the same time, with increasing of the gradient homogeneity of the concentration of dopant in enriched area increases.

To choose annealing time, it should be accounted decreasing of absolute value of gradient of concentration of dopant in neighborhood of interface between substrate and epitaxial layer with increasing of annealing time. Decreasing of the value of annealing time leads to decreasing of homogeneity of the concentration of dopant in enriched area (Figure 3a for diffusion doping of materials and Figure 3b for ion doping

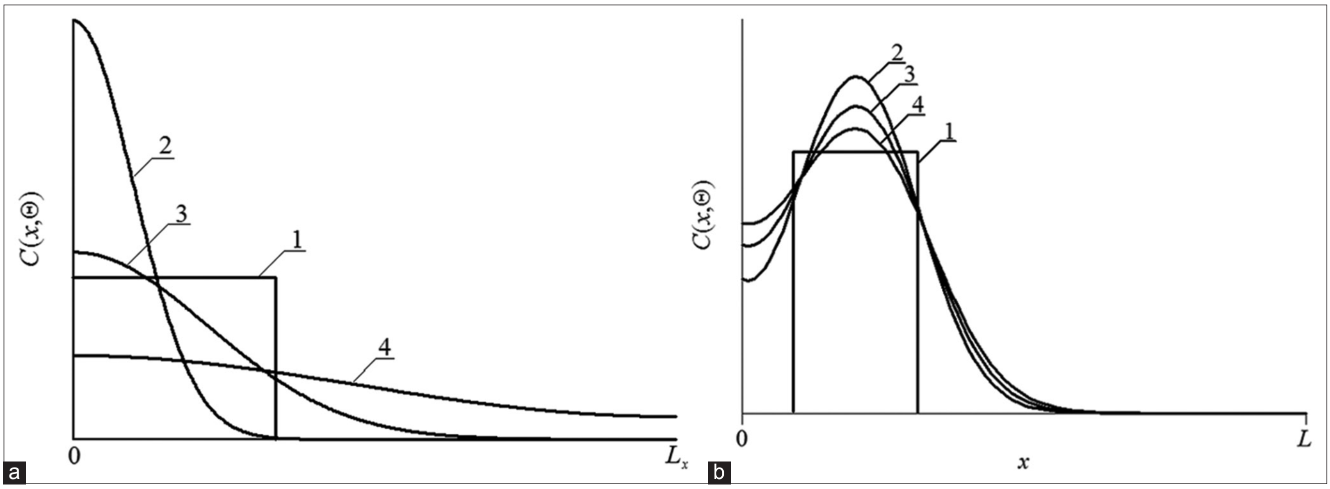


**Figure 3:** (a) Spatial distributions of dopant in heterostructure after dopant infusion. Curve 1 is idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time. Increasing of the number of curve corresponds to increasing of annealing time. (b) Spatial distributions of dopant in heterostructure after ion implantation. Curve 1 is idealized distribution of dopant. Curves 2–4 are real distributions of dopant for different values of annealing time. Increasing of the number of curve corresponds to increasing of annealing time

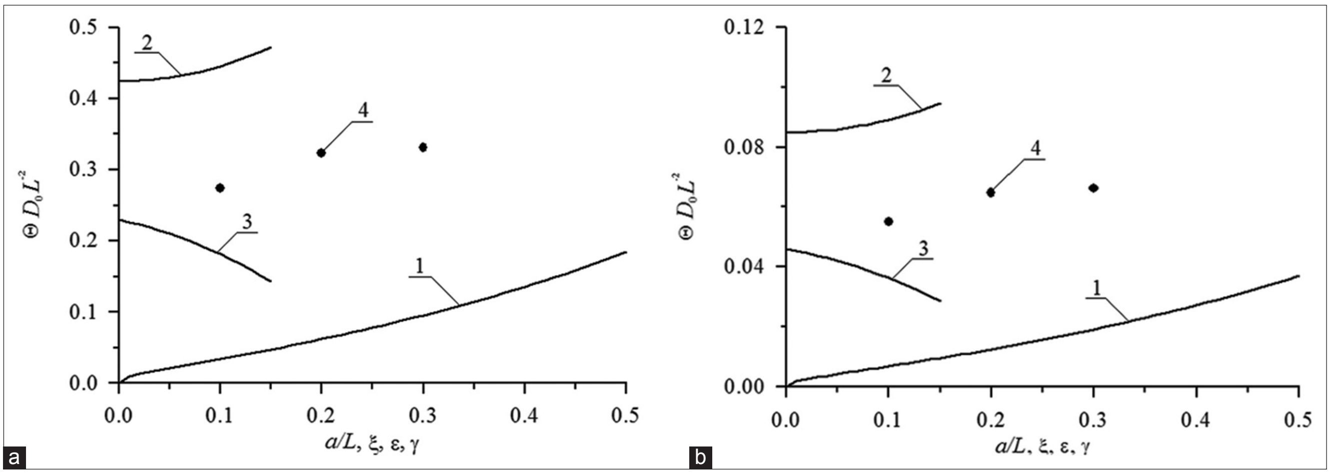


**Figure 4:** (a) Dependences of dimensionless optimal annealing time for doping by diffusion, which have been obtained by minimization of mean squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation *a/L* and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter $\varepsilon$ for *a/L*=1/2 and $\xi = \gamma = 0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter $\xi$ for *a/L* =1/2 and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter $\gamma$ for *a/L*=1/2 and $\varepsilon = \xi = 0$. (b) Dependences of dimensionless optimal annealing time for doping by ion implantation, which have been obtained by minimization of mean squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation *a/L* and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter *e* for *a/L*=1/2 and $\xi = \gamma = 0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter $\xi$ for *a/L* =1/2 and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter $\gamma$ for *a/L*=1/2 and $\varepsilon = \xi = 0$

of materials). Let us determine, compromise value of annealing time framework recently introduced criteria.[15-20] Framework the criteria, we approximate real distributions of the concentration of dopant by ideal rectangle distribution $\psi(x,y,z)$. Farther, we determine compromise value of annealing time by minimization of the mean squared error.

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left[ C(x,y,z,\Theta) - \psi(x,y,z) \right] d\,z\,d\,y\,d\,x. \tag{8}$$

Dependences of optimal annealing time are presented in Figure 4 for diffusion and ion types of doping, respectively. It should be noted that it is necessary to anneal radiation defects after ion implantation. One could find spreading of the concentration of distribution of dopant during this annealing. In the ideal case, distribution of dopant achieves appropriate interfaces between materials of heterostructure during annealing of radiation defects. If dopant did not achieve any interfaces during annealing of radiation defects, it is practicably to additionally anneal the dopant. In this situation, optimal value of additional annealing time of implanted dopant is smaller than annealing time of infused dopant. At the same time, ion type of doping gives us possibility to decrease mismatch-induced stress in heterostructure.[21]

## CONCLUSION

In this paper, we model redistribution of infused and implanted dopants during manufacture CMOP voltage differencing inverting buffered amplifier based on field-effect heterotransistors. Several recommendations to optimize manufacture the heterotransistors have been formulated. Analytical approach to model diffusion and ion types of doping with account concurrent changing of parameters in space and time has been introduced. At the same time, the approach gives us possibility to take into account nonlinearity of doping processes.

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