ISSN 2581-3463

AJMS
Asian Journal of Mathematical Sciences

RESEARCH ARTICLE

On Optimization of Manufacturing of a Two-level Current-mode Logic Gates in a Multiplexer

E. L. Pankratov[1,2]

[1]Department of Mathematical and Natural Sciences, Nizhny Novgorod State University, 23 Gagarin Avenue, Nizhny Novgorod 603950, Russia, [2]Department of Higher Mathematics, Nizhny Novgorod State Technical University, Nizhny Novgorod 603950, Russia

Received: 20-12-2019; Revised: 07-01-2020; Accepted: 20-01-2020

ABSTRACT

In this paper, we introduce an approach to increase the density of field-effect transistors framework a two-level current-mode logic gates in a multiplexer. Framework the approach we consider manufacturing the inverter in heterostructure with the specific configuration. Several required areas of the heterostructure should be doped by diffusion or ion implantation. After that, dopant and radiation defects should by annealed framework optimized scheme. We also consider an approach to decrease the value of mismatch-induced stress in the considered heterostructure. We introduce an analytical approach to analyze mass and heat transport in heterostructures during the manufacturing of integrated circuits with account mismatch-induced stress.

Key words: Increasing of element integration rate, optimization of manufacturing, two-level current-mode logic

INTRODUCTION

In the present time, several actual problems of the solid-state electronics (such as increasing of performance, reliability, and density of elements of integrated circuits: Diodes, field-effect, and bipolar transistors) are intensively solving.[1-6] To increase the performance of these devices, it is attracted an interest determination of materials with higher values of charge carriers mobility.[7-10] One way to decrease dimensions of elements of integrated circuits is manufacturing them in thin-film heterostructures.[3-5,11] In this case, it is possible to use inhomogeneity of heterostructure and necessary optimization of doping of electronic materials[12] and the development of epitaxial technology to improve these materials (including analysis of mismatch induced stress).[13-15] Alternative approaches to increase dimensions of integrated circuits are using laser and microwave types of annealing.[16-18]

Framework the paper, we introduce an approach to manufacture field-effect transistors. The approach gives a possibility to decrease their dimensions with increasing their density framework two-level current-mode logic gates in a multiplexer. We also consider the possibility to decrease mismatch-induced stress to decrease the quantity of defects, generated due to the stress. In this paper, we consider a heterostructure, which consists of a substrate and an epitaxial layer [Figure 1]. We also consider a buffer layer between the substrate and the epitaxial layer. The epitaxial layer includes itself several sections, which were manufactured using another materials. These sections have been doped by diffusion or ion implantation to manufacture the required types of conductivity (*p* or *n*). These areas became sources, drains, and gates.

Address for correspondence:
E. L. Pankratov
E-mail: elp2004@mail.ru

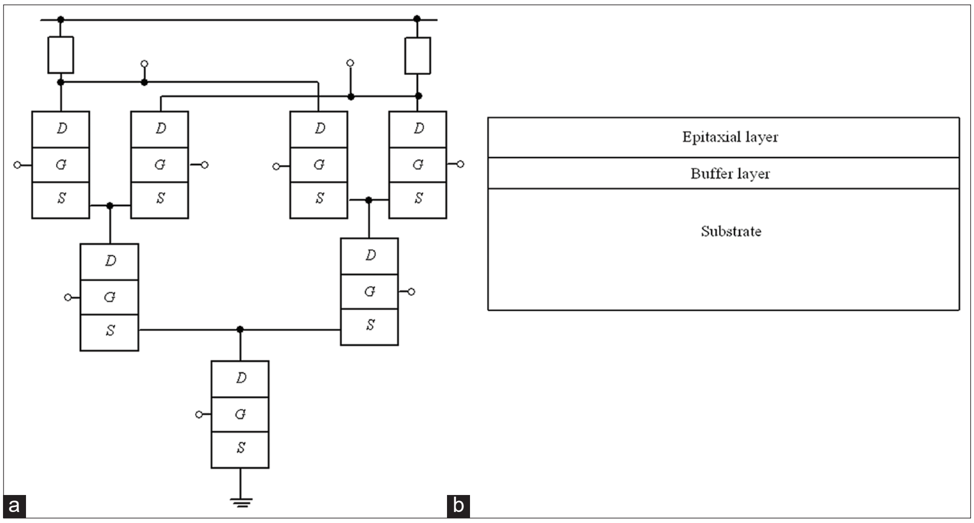

Figure 1: (a) Structure of the considered logic gates[19] (b) Heterostructure with a substrate, epitaxial layers, and buffer layer (view from side)

[Figure 1]. After this doping, it is required annealing of dopant and/or radiation defects. Main aim of the present paper is analysis of redistribution of dopant and radiation defects to determine conditions, which correspond to the decreasing of elements of the considered voltage reference and at the same time to increase their density. At the same time, we consider a possibility to decrease mismatch-induced stress.

METHOD OF SOLUTION

To solve our aim, we determine and analyzed the spatio-temporal distribution of the concentration of dopant in the considered heterostructure. We determine the distribution by solving the second Fick's law in the following form.[1,20-23]

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+$$
$$\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]+$$
$$\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]+ \tag{1}$$
$$+\frac{\partial}{\partial x}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\,\mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\,\mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\,\mu_2(x,y,z,t)}{\partial z}\right]$$

with boundary and initial conditions

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ C(x,y,z,0)=f_C(x,y,z),$$
$$\left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{x=L_y}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{x=L_z}=0.$$

Here, $C(x,y,z,t)$ is the spatio-temporal distribution of concentration of dopant; Ω is the atomic volume of dopant; ∇_s is the symbol of surficial gradient; $\int_0^{L_z}C(x,y,z,t)\,dz$ is the surficial concentration of dopant on interface between layers of heterostructure (in this situation we assume, that Z-axis is perpendicular to interface between layers of heterostructure); $\mu_1(x,y,z,t)$ and $\mu_2(x,y,z,t)$ are the chemical potential due to the presence of mismatch-induced stress and porosity of material; D and D_S are the coefficients of volumetric and surficial diffusions. Values of dopant diffusions coefficients depend on properties of materials of heterostructure, speed of heating and cooling of materials during annealing, and spatio-temporal distribution of concentration of dopant. Dependences of dopant diffusions coefficients on parameters could be approximated by the following relations.[24-26]

$$D_C = D_L(x,y,z,T)\left[1+\xi\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right],$$

$$D_S = D_{SL}(x,y,z,T)\left[1+\xi_S\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right] \quad (2)$$

Here, $D_L(x,y,z,T)$ and $D_{LS}(x,y,z,T)$ are the spatial (due to accounting all layers of heterostructure) and temperature (due to Arrhenius law) dependences of dopant diffusion coefficients; T is the temperature of annealing; $P(x,y,z,T)$ is the limit of solubility of dopant; parameter γ depends on properties of materials and could be integer in the following interval $\gamma \epsilon$;[1,3,24] $V(x,y,z,t)$ is the spatio-temporal distribution of concentration of radiation vacancies; V^* is the equilibrium distribution of vacancies. Concentration dependence of the dopant diffusion coefficient has been described in details in Gotra.[24] Spatio-temporal distributions of concentration of point radiation defects have been determined by solving the following system of equations.[20-23,25,26]

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]-k_{I,I}(x,y,z,T)\,I^2(x,y,z,t)-k_{I,V}(x,y,z,T)\times$$

$$I(x,y,z,t)V(x,y,z,t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]+$$

$$\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]+\frac{\partial}{\partial x}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}\right]+$$

$$\frac{\partial}{\partial y}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}\right] \quad (3)$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]-k_{V,V}(x,y,z,T)\,V^2(x,y,z,t)-k_{I,V}(x,y,z,T)\times$$

$$\times I(x,y,z,t)V(x,y,z,t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]+$$

$$\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]+\frac{\partial}{\partial x}\left[\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}\right]+$$

$$\frac{\partial}{\partial y}\left[\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}\right]$$

with boundary and initial conditions

$$\left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0 \tag{4}$$

$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$ $I(x,y,z,0)=f_I(x,y,z)$, $V(x,y,z,0)=fV$ (x,y,z), $V(x_1+V_nt,y_1+V_nt,z_1+V_nt,t)=V_\infty\left(1+\frac{2l\omega}{kT\sqrt{x_1^2+y_1^2+z_1^2}}\right)$.

Here, $I(x,y,z,t)$ is the spatio-temporal distribution of concentration of radiation interstitials; I^* is the equilibrium distribution of interstitials; $D_I(x,y,z,T)$, $D_V(x,y,z,T)$, $D_{IS}(x,y,z,T)$, and $D_{VS}(x,y,z,T)$ are the coefficients of volumetric and surficial diffusions of interstitials and vacancies, respectively; terms $V^2(x,y,z,t)$ and $I^2(x,y,z,t)$ correspond to generation of divacancies and diinterstitials, respectively (e.g. Vinetskiy and Kholodar[26] and appropriate references in this book); $k_{I,V}(x,y,z,T)$, $k_{I,I}(x,y,z,T)$, and $k_{V,V}(x,y,z,T)$ are the parameters of recombination of point radiation defects and generation of their complexes; k is the Boltzmann constant; $\omega=a^3$, a is the interatomic distance; l is the specific surface energy. To account porosity of buffer layers, we assume, that porous are approximately cylindrical with average values $r=\sqrt{x_1^2+y_1^2}$ and z_1 before annealing.[23] With time small pores decomposing on vacancies. The vacancies absorbing by larger pores.[27] With time large pores became larger due to absorbing the vacancies and became more spherical.[27] Distribution of concentration of vacancies in the heterostructure, existing due to porosity, could be determined by summing on all pores, i.e.,

$$V(x,y,z,t)=\sum_{i=0}^{l}\sum_{j=0}^{m}\sum_{k=0}^{n}V_p(x+i\alpha,y+j\beta,z+k\chi,t),\ R=\sqrt{x^2+y^2+z^2}.$$

Here, α,β, and χ are the average distances between centers of pores in directions x, y, and z; l, m, and n are the quantity of pores inappropriate directions.

Spatio-temporal distributions of divacancies $\Phi_V(x,y,z,t)$ and di-interstitials $\Phi_I(x,y,z,t)$ could be determined by solving the following system of equations.[25,26]

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+$$

$$\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+$$

$$\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}\right]+$$

$$k_I(x,y,z,T)I(x,y,z,t) \tag{5}$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+$$

$$\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+$$

$$\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}\right]+$$

$$k_V(x,y,z,T)V(x,y,z,t)$$

with boundary and initial conditions

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right|_{y=0}=0 \tag{6}$$

$$\left.\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$\Phi_I(x,y,z,0)=f_{\Phi I}(x,y,z)$, $\Phi_V(x,y,z,0)=f_{\Phi V}(x,y,z)$.

Here, $D_{\Phi I}(x,y,z,T)$, $D_{\Phi V}(x,y,z,T)$, $D_{\Phi IS}(x,y,z,T)$, and $D_{\Phi VS}(x,y,z,T)$ are the coefficients of volumetric and surficial diffusions of complexes of radiation defects; $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ are the parameters of decay of complexes of radiation defects.

Chemical potential μ_1 in Eq.(1) could be determined by the following relation[20]

$$\mu_1=E(z)\Omega\sigma_{ij}\,[u_{ij}(x,y,z,t)+u_{ji}(x,y,z,t)]/2, \tag{7}$$

where $E(z)$ is the Young modulus, σ_{ij} is the stress tensor; $u_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ is the deformation tensor; u_i and u_j are the components $u_x(x,y,z,t)$, $u_y(x,y,z,t)$, and $u_z(x,y,z,t)$ of the displacement vector $\vec{u}(x,y,z,t)$; x_i and x_j are the coordinate x, y, z. The eq. (3) could be transformed to the following form

$$\mu(x,y,z,t)=\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]\left\{\frac{1}{2}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]-\right.$$

$$\varepsilon_0\delta_{ij}+\frac{\sigma(z)\delta_{ij}}{1-2\sigma(z)}\left[\frac{\partial u_k(x,y,z,t)}{\partial x_k}-3\varepsilon_0\right]-K(z)\beta(z)\left[T(x,y,z,t)-T_0\right]\delta_{ij}\right\}\frac{\Omega}{2}E(z)$$

where σ is Poisson coefficient; $\varepsilon_0=(a_s-a_{EL})/a_{EL}$ is the mismatch parameter; a_s and a_{EL} are lattice distances of the substrate and the epitaxial layer; K is the modulus of uniform compression; β is the coefficient of thermal expansion; T_r is the equilibrium temperature, which coincides (for our case) with room temperature. Components of displacement vector could be obtained by solution of the following equations.[21]

$$\rho(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2}=\frac{\partial\sigma_{xx}(x,y,z,t)}{\partial x}+\frac{\partial\sigma_{xy}(x,y,z,t)}{\partial y}+\frac{\partial\sigma_{xz}(x,y,z,t)}{\partial z}$$

$$\rho(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\frac{\partial\sigma_{yx}(x,y,z,t)}{\partial x}+\frac{\partial\sigma_{yy}(x,y,z,t)}{\partial y}+\frac{\partial\sigma_{yz}(x,y,z,t)}{\partial z}$$

$$\rho(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\frac{\partial\sigma_{zx}(x,y,z,t)}{\partial x}+\frac{\partial\sigma_{zy}(x,y,z,t)}{\partial y}+\frac{\partial\sigma_{zz}(x,y,z,t)}{\partial z}$$

where $\sigma_{ij}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}-\frac{\delta_{ij}}{3}\frac{\partial u_k(x,y,z,t)}{\partial x_k}\right]+K(z)\delta_{ij}\times$

$\frac{\partial u_k(x,y,z,t)}{\partial x_k}-\beta(z)K(z)\left[T(x,y,z,t)-T_r\right]$, $\rho(z)$ is the density of materials of heterostructure, δ_{ij} Is the Kronecker symbol. With account, the relation for σ_{ij} last system of the equation could be written as

$$\rho(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial^2 u_x(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}\times$$

$$\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}+\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial z^2}\right]+\left[K(z)+\frac{E(z)}{3\left[1+\sigma(z)\right]}\right]\times$$

$$\frac{\partial^2 u_z(x,y,z,t)}{\partial x\partial z}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x}$$

$$\rho(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial y}\right]-\frac{\partial T(x,y,z,t)}{\partial y}\times$$

$$K(z)\beta(z)+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial u_y(x,y,z,t)}{\partial z}+\frac{\partial u_z(x,y,z,t)}{\partial y}\right]\right\}+\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}\times$$

$$\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+K(z)\right\}+\left\{K(z)-\frac{E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}+K(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}$$

$$\rho(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial^2 u_z(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial z}+\right.$$

$$\left.\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}\right]+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_x(x,y,z,t)}{\partial x}+\frac{\partial u_y(x,y,z,t)}{\partial y}+\frac{\partial u_x(x,y,z,t)}{\partial z}\right]\right\}+$$

$$\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[6\frac{\partial u_z(x,y,z,t)}{\partial z}-\frac{\partial u_x(x,y,z,t)}{\partial x}-\frac{\partial u_y(x,y,z,t)}{\partial y}-\frac{\partial u_z(x,y,z,t)}{\partial z}\right]\right\}-$$

$$K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z} \tag{8}$$

Conditions for the system of Equation (8) could be written in the form

$$\frac{\partial \vec{u}(0,y,z,t)}{\partial x}=0;\ \frac{\partial \vec{u}(L_x,y,z,t)}{\partial x}=0;\ \frac{\partial \vec{u}(x,0,z,t)}{\partial y}=0;\ \frac{\partial \vec{u}(x,L_y,z,t)}{\partial y}=0;$$

$$\frac{\partial \vec{u}(x,y,0,t)}{\partial z}=0;\ \frac{\partial \vec{u}(x,y,L_z,t)}{\partial z}=0;\ \vec{u}(x,y,z,0)=\vec{u}_0;\ \vec{u}(x,y,z,\infty)=\vec{u}_0.$$

We determine spatio-temporal distributions of concentrations of dopant and radiation defects by solving the Equations (1), (3), and (5) framework standard method of averaging of function corrections.[28] Previously, we transform the Equations (1), (3), and (5) to the following form with account initial distributions of the considered concentrations.

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+f_c(x,y,z)\delta(t)$$

$$\frac{\partial}{\partial x}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}\right]+$$

$$\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]+$$

$$\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right] \tag{1a}$$

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]+$$

$$\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]-k_{I,I}(x,y,z,T)I^2(x,y,z,t)-$$

$$k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+f_I(x,y,z)\delta(t) \tag{3a}$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]+$$

$$\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]-k_{V,V}(x,y,z,T)V^2(x,y,z,t)-$$

$$k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+f_V(x,y,z)\delta(t)$$

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+$$

$$\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+k_I(x,y,z,T)I(x,y,z,t)+$$

$$\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}\right]+$$

$$k_{I,I}(x,y,z,T)I^2(x,y,z,t)+f_{\Phi_I}(x,y,z)\delta(t) \tag{5a}$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+$$

$$\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+k_V(x,y,z,T)I(x,y,z,t)+$$

$$\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}\right]+$$

$$k_{V,V}(x,y,z,T)V^2(x,y,z,t)+f_{\Phi_V}(x,y,z)\delta(t).$$

Farther, we replace concentrations of dopant and radiation defects in the right sides of Equations (1a), (3a), and (5a) on their not yet known average values α_{1r}. In this situation, we obtain equations for the first-order approximations of the required concentrations in the following form

$$\frac{\partial C_1(x,y,z,t)}{\partial t}=\alpha_{1C}\Omega\frac{\partial}{\partial x}\left[z\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1C}\Omega\frac{\partial}{\partial y}\left[z\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\right]+$$

$$f_C(x,y,z)\delta(t)+\frac{\partial}{\partial x}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}\right] \tag{1b}$$

$$\frac{\partial I_1(x,y,z,t)}{\partial t}=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+\alpha_{1I}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+$$

$$\frac{\partial}{\partial x}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}\right]+$$

$$f_I(x,y,z)\delta(t)-\alpha_{1I}^2 k_{I,I}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T) \quad (3b)$$

$$\frac{\partial V_1(x,y,z,t)}{\partial t}=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1V}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+$$

$$\frac{\partial}{\partial x}\left[\frac{D_{VS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{VS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{VS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}\right]+$$

$$f_V(x,y,z)\delta(t)-\alpha_{1V}^2 k_{V,V}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T)$$

$$\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+$$

$$\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_I S}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}\right]+$$

$$f_{\Phi_I}(x,y,z)\delta(t)+k_I(x,y,z,T)I(x,y,z,t)+k_{I,I}(x,y,z,T)I^2(x,y,z,t) \quad (5b)$$

$$\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+$$

$$\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_V S}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}\right]+$$

$$f_{\Phi_V}(x,y,z)\delta(t)+k_V(x,y,z,T)V(x,y,z,t)+k_{V,V}(x,y,z,T)V^2(x,y,z,t).$$

Integration of the left and right sides of the Equations (1b), (3b), and (5b) on time gives us possibility to obtain relations for above approximation in the final form

$$C_1(x,y,z,t)=\alpha_{1C}\Omega\frac{\partial}{\partial x}\int_0^t D_{SL}(x,y,z,T)\frac{z}{kT}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$

$$\nabla_S\mu_1(x,y,z,\tau)\left[1+\frac{\xi_S\alpha_{1C}^\gamma}{P^\gamma(x,y,z,T)}\right]d\tau\Bigg\}+\alpha_{1C}\frac{\partial}{\partial y}\int_0^t D_{SL}(x,y,z,T)\left[1+\frac{\xi_S\alpha_{1C}^\gamma}{P^\gamma(x,y,z,T)}\right]+$$

$$\Omega\nabla_S\mu_1(x,y,z,\tau)\frac{z}{kT}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]d\tau+\frac{\partial}{\partial x}\int_0^t\frac{D_{CS}}{\bar V kT}\times$$

$$\frac{\partial\mu_2(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t\frac{D_{CS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t\frac{D_{CS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial z}d\tau+$$

$$f_C(x,y,z) \quad (1c)$$

$$I_1(x,y,z,t)=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau+\alpha_{1I}z\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+$$

$$\frac{\partial}{\partial x}\int_0^t \frac{D_{IS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t \frac{D_{IS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t \frac{D_{IS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}d\tau+$$

$$f_I(x,y,z)-\alpha_{1I}^2\int_0^t k_{I,I}(x,y,z,T)\,d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau \tag{3c}$$

$$V_1(x,y,z,t)=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\int_0^t \frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+\alpha_{1V}z\Omega\frac{\partial}{\partial y}\int_0^t \frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+$$

$$\frac{\partial}{\partial x}\int_0^t \frac{D_{VS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t \frac{D_{VS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t \frac{D_{VS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}d\tau+$$

$$f_V(x,y,z)-\alpha_{1V}^2\int_0^t k_{V,V}(x,y,z,T)\,d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau$$

$$\Phi_{1I}(x,y,z,t)=\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\int_0^t \frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+\Omega\frac{\partial}{\partial x}\int_0^t \frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau\times$$

$$\alpha_{1\Phi_I}z+f_{\Phi_I}(x,y,z)+\frac{\partial}{\partial x}\int_0^t \frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t \frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,\tau)}{\partial y}d\tau+$$

$$\frac{\partial}{\partial z}\int_0^t \frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,\tau)}{\partial z}d\tau+\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)\,d\tau+$$

$$+\int_0^t k_{I,I}(x,y,z,T)I^2(x,y,z,\tau)\,d\tau \tag{5c}$$

$$\Phi_{1V}(x,y,z,t)=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\int_0^t \frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+\Omega\frac{\partial}{\partial x}\int_0^t \frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau\times$$

$$\alpha_{1\Phi_V}z+f_{\Phi_V}(x,y,z)+\frac{\partial}{\partial x}\int_0^t \frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t \frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,\tau)}{\partial y}d\tau+$$

$$\frac{\partial}{\partial z}\int_0^t \frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,\tau)}{\partial z}d\tau+\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)\,d\tau+$$

$$\int_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,\tau)\,d\tau$$

We determine the average values of the first-order approximations of concentrations of dopant and radiation defects by the following standard relation.[28]

$$\alpha_{1\rho}=\frac{1}{\Theta L_x L_y L_z}\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\rho_1(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t \tag{9}$$

Substitution of the relations Equations (1c), (3c), and (5c) into relation Equation (9) gives us the possibility to obtain required average values in the following form

$$\alpha_{1C}=\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_C(x,y,z)\,d\,z\,d\,y\,d\,x,\ \alpha_{1I}=\sqrt{\frac{(a_3+A)^2}{4a_4^2}-4\left(B+\frac{\Theta a_3 B+\Theta^2 L_x L_y L_z a_1}{a_4}\right)}-$$

$$-\frac{a_3+A}{4a_4},\ \alpha_{1V}=\frac{1}{S_{IV00}}\left[\frac{\Theta}{\alpha_{1I}}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx-\alpha_{1I}S_{II00}-\Theta L_xL_yL_z\right],$$

where $S_{\rho\rho\,ij}=\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} k_{\rho,\rho}(x,y,z,T)\,I_1^i(x,y,z,t)\,V_1^j(x,y,z,t)\,dz\,dy\,dx\,dt$, $a_4=S_{II00}\times$

$$\left(S_{IV00}^2-S_{II00}S_{VV00}\right),\ a_3=S_{IV00}S_{II00}+S_{IV00}^2-S_{II00}S_{VV00},\ a_2=\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_V(x,y,z)\,dz\,dy\,dx\times$$

$$S_{IV00}S_{IV00}^2+S_{IV00}\Theta L_x^2L_y^2L_z^2+2S_{VV00}S_{II00}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx-\Theta L_x^2L_y^2L_z^2S_{VV00}-$$

$$S_{IV00}^2\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx,\ a_1=S_{IV00}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx,\ a_0=S_{VV00}\times$$

$$\left[\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx\right]^2,\ A=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}},\ B=\frac{\Theta a_2}{6a_4}+\sqrt[3]{\sqrt{q^2+p^3}-q}-$$

$$\sqrt[3]{\sqrt{q^2+p^3}+q},\ q=\frac{\Theta^3a_2}{24a_4^2}\left(4a_0-\Theta L_xL_yL_z\frac{a_1a_3}{a_4}\right)-\Theta^2\frac{a_0}{8a_4^2}\left(4\Theta a_2-\Theta^2\frac{a_3^2}{a_4}\right)-$$

$$\frac{\Theta^3a_2^3}{54a_4^3}-L_x^2L_y^2L_z^2\frac{\Theta^4a_1^2}{8a_4^2},\ p=\Theta^2\frac{4a_0a_4-\Theta L_xL_yL_za_1a_3}{12a_4^2}-\frac{\Theta a_2}{18a_4},$$

$$\alpha_{1\Phi_I}=\frac{R_{I1}}{\Theta L_xL_yL_z}+\frac{S_{II20}}{\Theta L_xL_yL_z}+\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_{\Phi_I}(x,y,z)\,dz\,dy\,dx$$

$$\alpha_{1\Phi_V}=\frac{R_{V1}}{\Theta L_xL_yL_z}+\frac{S_{VV20}}{\Theta L_xL_yL_z}+\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_{\Phi_V}(x,y,z)\,dz\,dy\,dx,$$

where $R_{\rho\,i}=\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} k_I(x,y,z,T)\,I_1^i(x,y,z,t)\,dz\,dy\,dx\,dt$.

We determine approximations of the second and higher orders of concentrations of dopant and radiation defects framework standard iterative procedure of the method of averaging of function corrections.[28] Framework this procedure to determine approximations of the n-th order of concentrations of dopant and radiation defects, we replace the required concentrations in the Equations (1c), (3c), and (5c) on the following sum $\alpha_{n\rho}+\rho_{n-1}(x,y,z,t)$. The replacement leads to the following transformation of the appropriate equations

$$\frac{\partial C_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left(\left\{1+\xi\frac{\left[\alpha_{2C}+C_1(x,y,z,t)\right]^{\gamma}}{P^{\gamma}(x,y,z,T)}\right\}\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{\left(V^*\right)^2}\right]\times\right.$$

$$\left.D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial x}\right)+\frac{\partial}{\partial y}\left(\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{\left(V^*\right)^2}\right]\frac{\partial C_1(x,y,z,t)}{\partial y}\times\right.$$

$$D_L(x,y,z,T)\left\{1+\xi\frac{\left[\alpha_{2C}+C_1(x,y,z,t)\right]^{\gamma}}{P^{\gamma}(x,y,z,T)}\right\}\Bigg)+\frac{\partial}{\partial z}\Bigg(\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{\left(V^*\right)^2}\right]\times$$

$$D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial z}\left\{1+\xi\frac{\left[\alpha_{2C}+C_1(x,y,z,t)\right]^{\gamma}}{P^{\gamma}(x,y,z,T)}\right\}\Bigg)+f_C(x,y,z)\delta(t)+$$

$$\frac{\partial}{\partial x}\left[\frac{D_{CS}}{\bar V kT}\frac{\partial\,\mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{CS}}{\bar V kT}\frac{\partial\,\mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{CS}}{\bar V kT}\frac{\partial\,\mu_2(x,y,z,t)}{\partial z}\right]+$$

$$\Omega\frac{\partial}{\partial x}\left\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2C}+C(x,y,W,t)\right]dW\right\}+$$

$$\Omega\frac{\partial}{\partial y}\left\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2C}+C(x,y,W,t)\right]dW\right\} \tag{1d}$$

$$\frac{\partial\,I_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial z}\right]-k_{I,I}(x,y,z,T)\left[\alpha_{1I}+I_1(x,y,z,t)\right]^2-k_{I,V}(x,y,z,T)\times$$

$$\left[\alpha_{1I}+I_1(x,y,z,t)\right]\left[\alpha_{1V}+V_1(x,y,z,t)\right]+\Omega\frac{\partial}{\partial x}\left\{\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,t)\right]dW\times\right.$$

$$\left.\frac{D_{IS}}{kT}\right\}+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,t)\right]dW\right\}+\frac{\partial}{\partial x}\int\limits_0^t\frac{\partial\mu_2(x,y,z,t)}{\partial x}\times$$

$$\frac{D_{IS}}{\bar V kT}d\tau+\frac{\partial}{\partial y}\int\limits_0^t\frac{D_{IS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}d\tau+\frac{\partial}{\partial z}\int\limits_0^t\frac{D_{IS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}d\tau \tag{3d}$$

$$\frac{\partial\,V_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial z}\right]-k_{V,V}(x,y,z,T)\left[\alpha_{1V}+V_1(x,y,z,t)\right]^2-k_{I,V}(x,y,z,T)\times$$

$$\left[\alpha_{1I}+I_1(x,y,z,t)\right]\left[\alpha_{1V}+V_1(x,y,z,t)\right]+\Omega\frac{\partial}{\partial x}\left\{\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,t)\right]dW\times\right.$$

$$\left.\frac{D_{VS}}{kT}\right\}+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,t)\right]dW\right\}+\frac{\partial}{\partial x}\int\limits_0^t\frac{\partial\mu_2(x,y,z,t)}{\partial x}\times$$

$$\frac{D_{VS}}{\bar V kT}d\tau+\frac{\partial}{\partial y}\int\limits_0^t\frac{D_{VS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}d\tau+\frac{\partial}{\partial z}\int\limits_0^t\frac{D_{VS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}d\tau$$

$$\frac{\partial\,\Phi_{2I}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,t)}{\partial y}\right]+$$

$$\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}\left(x,y,W,t\right)\right]dW\right\}+k_{I,I}\left(x,y,z,T\right)I^2\left(x,y,z,t\right)+$$

$$\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}\left(x,y,W,t\right)\right]dW\right\}+k_I\left(x,y,z,T\right)I\left(x,y,z,t\right)+$$

$$\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial z}\right]+$$

$$\frac{\partial}{\partial z}\left[D_{\Phi_I}\left(x,y,z,T\right)\frac{\partial\Phi_{1I}\left(x,y,z,t\right)}{\partial z}\right]+f_{\Phi_I}\left(x,y,z\right)\delta\left(t\right)\tag{5d}$$

$$\frac{\partial\Phi_{2V}\left(x,y,z,t\right)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}\left(x,y,z,T\right)\frac{\partial\Phi_{1V}\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}\left(x,y,z,T\right)\frac{\partial\Phi_{1V}\left(x,y,z,t\right)}{\partial y}\right]+$$

$$\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}\left(x,y,W,t\right)\right]dW\right\}+k_{V,V}\left(x,y,z,T\right)V^2\left(x,y,z,t\right)+$$

$$\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}\left(x,y,W,t\right)\right]dW\right\}+k_V\left(x,y,z,T\right)V\left(x,y,z,t\right)+$$

$$\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial z}\right]+$$

$$\frac{\partial}{\partial z}\left[D_{\Phi_V}\left(x,y,z,T\right)\frac{\partial\Phi_{1V}\left(x,y,z,t\right)}{\partial z}\right]+f_{\Phi_V}\left(x,y,z\right)\delta\left(t\right).$$

Integration of the left and the right sides of Equations (1d), (3d), and (5d) gives us possibility to obtain relations for the required concentrations in the final form

$$C_2\left(x,y,z,t\right)=\frac{\partial}{\partial x}\int\limits_0^t\left\{1+\xi\frac{\left[\alpha_{2C}+C_1\left(x,y,z,\tau\right)\right]^\gamma}{P^\gamma\left(x,y,z,T\right)}\right\}\left[1+\varsigma_1\frac{V\left(x,y,z,\tau\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,\tau\right)}{\left(V^*\right)^2}\right]\times$$

$$D_L\left(x,y,z,T\right)\frac{\partial C_1\left(x,y,z,\tau\right)}{\partial x}d\tau+\frac{\partial}{\partial y}\int\limits_0^t D_L\left(x,y,z,T\right)\left[1+\varsigma_1\frac{V\left(x,y,z,\tau\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,\tau\right)}{\left(V^*\right)^2}\right]\times$$

$$\frac{\partial C_1\left(x,y,z,\tau\right)}{\partial y}\left\{1+\xi\frac{\left[\alpha_{2C}+C_1\left(x,y,z,t\right)\right]^\gamma}{P^\gamma\left(x,y,z,T\right)}\right\}+\frac{\partial}{\partial z}\int\limits_0^t\left[1+\varsigma_1\frac{V\left(x,y,z,\tau\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,\tau\right)}{\left(V^*\right)^2}\right]\times$$

$$D_L\left(x,y,z,T\right)\frac{\partial C_1\left(x,y,z,\tau\right)}{\partial z}\left\{1+\xi\frac{\left[\alpha_{2C}+C_1\left(x,y,z,\tau\right)\right]^\gamma}{P^\gamma\left(x,y,z,T\right)}\right\}d\tau+f_C\left(x,y,z\right)+$$

$$\Omega\frac{\partial}{\partial x}\int\limits_0^t\frac{D_S}{kT}\nabla_S\mu\left(x,y,z,\tau\right)\int\limits_0^{L_z}\left[\alpha_{2C}+C_1\left(x,y,W,\tau\right)\right]dW\,d\tau+\frac{\partial}{\partial y}\int\limits_0^t\nabla_S\mu\left(x,y,z,\tau\right)\times$$

$$\Omega\frac{D_S}{kT}\int\limits_0^{L_z}\left[\alpha_{2C}+C_1\left(x,y,W,\tau\right)\right]dW\,d\tau+\frac{\partial}{\partial x}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial x}\right]+$$

$$\frac{\partial}{\partial y}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}\right] \tag{1e}$$

$$I_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial y}d\tau+$$

$$\frac{\partial}{\partial z}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial z}d\tau-\int_0^t k_{I,I}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]^2 d\tau-$$

$$\int_0^t k_{I,V}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]d\tau+\frac{\partial}{\partial x}\int_0^t \nabla_S\mu(x,y,z,\tau)\times$$

$$\Omega\frac{D_{IS}}{kT}\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]dW\,d\tau+\frac{\partial}{\partial y}\int_0^t \nabla_S\mu(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]\times$$

$$\Omega\frac{D_{IS}}{kT}dW\,d\tau+\frac{\partial}{\partial x}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}\right]+f_I(x,y,z) \tag{3e}$$

$$V_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau+$$

$$\frac{\partial}{\partial z}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial z}d\tau-\int_0^t k_{V,V}(x,y,z,T)\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]^2 d\tau-$$

$$\int_0^t k_{I,V}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]d\tau+\frac{\partial}{\partial x}\int_0^t \nabla_S\mu(x,y,z,\tau)\times$$

$$\Omega\frac{D_{VS}}{kT}\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]dW\,d\tau+\frac{\partial}{\partial y}\int_0^t \nabla_S\mu(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]\times$$

$$\Omega\frac{D_{VS}}{kT}dW\,d\tau+\frac{\partial}{\partial x}\left[\frac{D_{VS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{VS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[\frac{D_{VS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}\right]+f_V(x,y,z)$$

$$\Phi_{2I}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t \frac{\partial \Phi_{1I}(x,y,z,\tau)}{\partial y}\times$$

$$D_{\Phi_I}(x,y,z,T)d\tau+\frac{\partial}{\partial z}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,\tau)}{\partial z}d\tau+\Omega\frac{\partial}{\partial x}\int_0^t \nabla_S\mu(x,y,z,\tau)\times$$

$$\frac{D_{\Phi_I S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t \frac{D_{\Phi_I S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\times$$

$$\nabla_S\mu\left(x,y,z,\tau\right)d\tau+\int_0^t k_{I,I}\left(x,y,z,T\right)I^2\left(x,y,z,\tau\right)d\tau+\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{\bar V kT}\frac{\partial\mu_2\left(x,y,z,\tau\right)}{\partial x}d\tau+$$

$$\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_I S}}{\bar V kT}\frac{\partial\mu_2\left(x,y,z,\tau\right)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t\frac{D_{\Phi_I S}}{\bar V kT}\frac{\partial\mu_2\left(x,y,z,\tau\right)}{\partial z}d\tau+f_{\Phi_I}\left(x,y,z\right)+$$

$$\int_0^t k_I\left(x,y,z,T\right)I\left(x,y,z,\tau\right)d\tau \tag{5e}$$

$$\Phi_{2V}\left(x,y,z,t\right)=\frac{\partial}{\partial x}\int_0^t D_{\Phi_V}\left(x,y,z,T\right)\frac{\partial\Phi_{1V}\left(x,y,z,\tau\right)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t\frac{\partial\Phi_{1V}\left(x,y,z,\tau\right)}{\partial y}\times$$

$$D_{\Phi_V}\left(x,y,z,T\right)d\tau+\frac{\partial}{\partial z}\int_0^t D_{\Phi_V}\left(x,y,z,T\right)\frac{\partial\Phi_{1V}\left(x,y,z,\tau\right)}{\partial z}d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\mu\left(x,y,z,\tau\right)\times$$

$$\frac{D_{\Phi_V S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}\left(x,y,W,\tau\right)\right]dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_V S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}\left(x,y,W,\tau\right)\right]dW\times$$

$$\nabla_S\mu\left(x,y,z,\tau\right)d\tau+\int_0^t k_{V,V}\left(x,y,z,T\right)V^2\left(x,y,z,\tau\right)d\tau+\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{\bar V kT}\frac{\partial\mu_2\left(x,y,z,\tau\right)}{\partial x}d\tau+$$

$$\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_V S}}{\bar V kT}\frac{\partial\mu_2\left(x,y,z,\tau\right)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t\frac{D_{\Phi_V S}}{\bar V kT}\frac{\partial\mu_2\left(x,y,z,\tau\right)}{\partial z}d\tau+f_{\Phi_V}\left(x,y,z\right)+$$

$$\int_0^t k_V\left(x,y,z,T\right)V\left(x,y,z,\tau\right)d\tau.$$

Average values of the second-order approximations of required approximations using the following standard relation.[28]

$$\alpha_{2\rho}=\frac{1}{\Theta L_xL_yL_z}\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[\rho_2\left(x,y,z,t\right)-\rho_1\left(x,y,z,t\right)\right]dz\,dy\,dx\,dt \tag{10}$$

Substitution of the relations Equations (1e), (3e), and (5e) into relation Equation (10) gives us the possibility to obtain relations for required average values $\alpha_{2\rho}$.

α_{2C}=0, $\alpha_{2\Phi I}$=0, $\alpha_{2\Phi V}$=0, $\alpha_{2V}=\sqrt{\frac{\left(b_3+E\right)^2}{4b_4^2}-4\left(F+\frac{\Theta a_3F+\Theta^2L_xL_yL_zb_1}{b_4}\right)}-\frac{b_3+E}{4b_4}$,

$$\alpha_{2I}=\frac{C_V-\alpha_{2V}^2S_{VV00}-\alpha_{2V}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)-S_{VV02}-S_{IV11}}{S_{IV01}+\alpha_{2V}S_{IV00}},$$

where $b_4=\frac{1}{\Theta L_xL_yL_z}S_{IV00}^2S_{VV00}-\frac{1}{\Theta L_xL_yL_z}S_{VV00}^2S_{II00}$, $b_3=-\frac{S_{II00}S_{VV00}}{\Theta L_xL_yL_z}(2S_{VV01}+S_{IV10}+$

$\Theta L_xL_yL_z)+\frac{S_{IV00}S_{VV00}}{\Theta L_xL_yL_z}\left(S_{IV01}+2S_{II10}+S_{IV01}+\Theta L_xL_yL_z\right)+\frac{S_{IV00}^2}{\Theta L_xL_yL_z}(2S_{VV01}+S_{IV10}+$

$\Theta L_xL_yL_z)-\frac{S_{IV00}^2S_{IV10}}{\Theta^3L_x^3L_y^3L_z^3}$, $b_2=\frac{S_{II00}S_{VV00}}{\Theta L_xL_yL_z}\left(S_{VV02}+S_{IV11}+C_V\right)-\left(S_{IV10}-2S_{VV01}+\Theta L_xL_y\times\right.$

$$L_z\big)^2+\frac{S_{IV01}S_{VV00}}{\Theta L_xL_yL_z}\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right)+\frac{S_{IV00}}{\Theta L_xL_yL_z}\left(S_{IV01}+2S_{II10}+2S_{IV01}+\Theta L_xL_y\times\right.$$

$$\left.L_z\right)\left(2S_{VV01}+\Theta L_xL_yL_z+S_{IV10}\right)-\frac{S_{IV00}^2}{\Theta L_xL_yL_z}\left(C_V-S_{VV02}-S_{IV11}\right)+\frac{C_IS_{IV00}^2}{\Theta^2L_x^2L_y^2L_z^2}-\frac{2S_{IV10}}{\Theta L_xL_yL_z}\times$$

$$S_{IV00}S_{IV01},\; b_1=S_{II00}\frac{S_{IV11}+S_{VV02}+C_V}{\Theta L_xL_yL_z}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)+\frac{S_{IV01}}{\Theta L_xL_yL_z}\left(\Theta L_xL_y\times\right.$$

$$\left.L_z+2S_{II10}+S_{IV01}\right)\left(2S_{VV01}+S_{IV10}+\Theta LL_yL_z\right)-\frac{S_{IV10}S_{IV01}^2}{\Theta L_xL_yL_z}-\frac{S_{IV00}}{\Theta L_xL_yL_z}\left(3S_{IV01}+2S_{II10}+\right.$$

$$\left.\Theta L_xL_yL_z\right)\left(C_V-S_{VV02}-S_{IV11}\right)+2C_IS_{IV00}S_{IV01},\; b_0=\frac{S_{II00}}{\Theta L_xL_yL_z}\left(S_{IV00}+S_{VV02}\right)^2-\frac{S_{IV01}}{L_xL_yL_z}\times$$

$$\frac{1}{\Theta}\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right)\left(C_V-S_{VV02}-S_{IV11}\right)+2C_IS_{IV01}^2-S_{IV01}\frac{C_V-S_{VV02}-S_{IV11}}{\Theta L_xL_yL_z}\times$$

$$\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right),C_I=\frac{\alpha_{1I}\alpha_{1V}}{\Theta L_xL_yL_z}S_{IV00}+\frac{\alpha_{1I}^2S_{II00}}{\Theta L_xL_yL_z}-\frac{S_{II20}S_{II20}}{\Theta L_xL_yL_z}-\frac{S_{IV11}}{\Theta L_xL_yL_z},$$

$$C_V=\alpha_{1I}\alpha_{1V}S_{IV00}+\alpha_{1V}^2S_{VV00}-S_{VV02}-S_{IV11},\; E=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}},\; F=\frac{\Theta a_2}{6a_4}+$$

$$\sqrt[3]{\sqrt{r^2+s^3}-r}-\sqrt[3]{\sqrt{r^2+s^3}+r},\; r=\frac{\Theta^3b_2}{24b_4^2}\left(4b_0-\Theta L_xL_yL_z\frac{b_1b_3}{b_4}\right)-\frac{\Theta^3b_2^3}{54b_4^3}-b_0\frac{\Theta^2}{8b_4^2}\times$$

$$\left(4\Theta b_2-\Theta^2\frac{b_3^2}{b_4}\right)-L_x^2L_y^2L_z^2\frac{\Theta^4b_1^2}{8b_4^2},\; s=\Theta^2\frac{4b_0b_4-\Theta L_xL_yL_zb_1b_3}{12b_4^2}-\frac{\Theta b_2}{18b_4}.$$

Farther, we determine solutions of Equation (8), i.e., components of the displacement vector. To determine the first-order approximations of the considered components framework method of averaging of function corrections, we replace the required functions in the right sides of the equations by their not yet known average values ai. The substitution leads to the following result

$$\rho(z)\frac{\partial^2u_{1x}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x},\;\rho(z)\frac{\partial^2u_{1y}(x,y,z,t)}{\partial t^2}=$$

$$-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y},\;\rho(z)\frac{\partial^2u_{1z}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Integration of the left and the right sides of the above relations on time t leads to the following result

$$u_{1x}(x,y,z,t)=u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^t\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta,$$

$$u_{1y}(x,y,z,t)=u_{0y}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^t\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta,$$

$$u_{1z}(x,y,z,t)=u_{0z}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^{t}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta.$$

Approximations of the second and higher orders of components of displacement vector could be determined using standard replacement of the required components on the following sums $\alpha_i+u_i(x,y,z,t)$.[28] The replacement leads to the following result

$$\rho(z)\frac{\partial^2 u_{2x}(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\times\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}+$$

$$\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial z^2}\right]-\frac{\partial T(x,y,z,t)}{\partial x}\times$$

$$K(z)\beta(z)+\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x\partial z}$$

$$\rho(z)\frac{\partial^2 u_{2y}(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial y}\right]-\frac{\partial T(x,y,z,t)}{\partial y}\times$$

$$K(z)\beta(z)+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_{1y}(x,y,z,t)}{\partial z}+\frac{\partial u_{1z}(x,y,z,t)}{\partial y}\right]\right\}+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}\times$$

$$\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}+\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}+K(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}$$

$$\rho(z)\frac{\partial^2 u_{2z}(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial z}+\right.$$

$$\left.\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}\right]+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_{1x}(x,y,z,t)}{\partial x}+\frac{\partial u_{1y}(x,y,z,t)}{\partial y}+\frac{\partial u_{1x}(x,y,z,t)}{\partial z}\right]\right\}+$$

$$\frac{E(z)}{6[1+\sigma(z)]}\frac{\partial}{\partial z}\left[6\frac{\partial u_{1z}(x,y,z,t)}{\partial z}-\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\right]-$$

$$\left.\left.\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\right]\right\}\frac{E(z)}{1+\sigma(z)}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Integration of the left and right sides of the above relations on time t leads to the following result

$$u_{2x}(x,y,z,t)=\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x^2}\int_0^{t}\int_0^{\vartheta} u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{1}{\rho(z)}\left\{K(z)-\right.$$

$$\left.\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\partial y}\int_0^{t}\int_0^{\vartheta} u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{E(z)}{2\rho(z)}\left[\frac{\partial^2}{\partial y^2}\int_0^{t}\int_0^{\vartheta} u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$

$$\frac{\partial^2}{\partial z^2}\int_0^t\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\frac{1}{1+\sigma(z)}+\frac{1}{\rho(z)}\frac{\partial^2}{\partial x\partial z}\int_0^t\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\,\Bigg\{K(z)+$$

$$\frac{E(z)}{3\left[1+\sigma(z)\right]}\Bigg\}-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\times$$

$$\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6\left[1+\sigma(z)\right]}\right\}-\left\{K(z)-\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}\frac{\partial^2}{\partial x\partial y}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\times$$

$$\frac{1}{\rho(z)}-\frac{E(z)}{2\rho(z)\left[1+\sigma(z)\right]}\left[\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial z^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]-$$

$$\frac{1}{\rho(z)}\left\{K(z)+\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}\frac{\partial^2}{\partial x\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\times$$

$$\times\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta$$

$$u_{2y}(x,y,z,t)=\frac{E(z)}{2\rho(z)\left[1+\sigma(z)\right]}\left[\frac{\partial^2}{\partial x^2}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\times$$

$$\frac{1}{1+\sigma(z)}+\frac{K(z)}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{1}{\rho(z)}\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+K(z)\right\}\times$$

$$\frac{\partial^2}{\partial y^2}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\Bigg\{\frac{E(z)}{1+\sigma(z)}\Bigg[\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+$$

$$\frac{\partial}{\partial y}\int_0^t\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\Bigg\}-K(z)\frac{\beta(z)}{\rho(z)}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-\Bigg\{\frac{E(z)}{6\left[1+\sigma(z)\right]}-$$

$$K(z)\Bigg\}\frac{1}{\rho(z)}\frac{\partial^2}{\partial y\partial z}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{E(z)}{2\rho(z)}\Bigg[\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+$$

$$\frac{\partial^2}{\partial x\partial y}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\frac{1}{1+\sigma(z)}-K(z)\frac{\beta(z)}{\rho(z)}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{K(z)}{\rho(z)}\times$$

$$\frac{\partial^2}{\partial x\partial y}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+$$

$$K(z)\Bigg\}-\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}\times$$

$$\frac{1}{2\rho(z)}-\frac{1}{\rho(z)}\left\{K(z)-\frac{E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial^2}{\partial y\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0y}$$

$$u_z(x,y,z,t)=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$

$$\frac{\partial^2}{\partial x \partial z}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta + \frac{\partial^2}{\partial y \partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\frac{1}{\rho(z)} + \frac{1}{\rho(z)}\times$$

$$\frac{\partial}{\partial z}\Bigg\{K(z)\Bigg[\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta + \frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta +$$

$$\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\Bigg\} + \frac{1}{6\rho(z)}\frac{\partial}{\partial z}\Bigg\{\frac{E(z)}{1+\sigma(z)}\Bigg[6\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta -$$

$$\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta - \frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta - \frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\Bigg]\Bigg\} -$$

$$K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta + u_{0z}.$$

Framework this paper, we determine the concentration of dopant, concentrations of radiation defects, and components of displacement vector using the second-order approximation framework method of averaging of function corrections. This approximation is usually enough good approximation to make qualitative analysis and to obtain some quantitative results. All obtained results have been checked by comparison with the results of numerical simulations.

DISCUSSION

In this section, we analyzed the dynamics of redistributions of dopant and radiation defects during annealing and under the influence of mismatch-induced stress and modification of porosity. Typical distributions of concentrations of infused dopant in heterostructures.

Increasing of number of the curve corresponds to increasing of annealing time is presented on Figures 2 and 3 for diffusion and ion types of doping, respectively. These distributions have been calculated for the case, when the value of dopant diffusion coefficient in doped area is larger than in the nearest areas. The figures show that inhomogeneity of heterostructure gives us the possibility to increase the compactness of concentrations of dopants and at the same time to

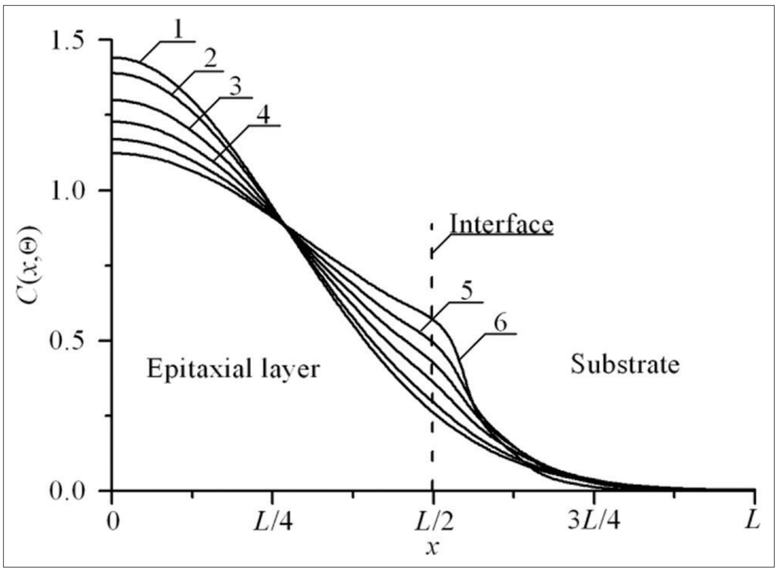

Figure 2: Distributions of concentration of infused dopant in heterostructure from Figure 1 in direction, which is perpendicular to the interface between epitaxial layer substrate. Increasing of number of the curve corresponds to increasing of difference between values of the dopant diffusion coefficient in layers of heterostructure under condition, when value of dopant diffusion coefficient in epitaxial layer is larger than the value of dopant diffusion coefficient in the substrate

increase the homogeneity of dopant distribution in doped part of the epitaxial layer. However, framework this approach of manufacturing of bipolar transistor is necessary to optimize annealing of dopant and/or radiation defects. Reason for, this optimization is the following. If annealing time is small, the dopant did not achieve any interfaces between materials of the heterostructure. In this situation, one cannot find any modifications of the distribution of the concentration of dopant. If annealing time is large, the distribution of concentration of dopant is too homogenous. We optimize the annealing time framework recently introduces approach.[29-37] Framework this criterion, we approximate real distribution of concentration of dopant by step-wise function [Figure 4 and 5]. Farther, we determine optimal values of annealing time by minimization of the following mean-squared error

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left[C(x,y,z,\Theta) - \psi(x,y,z) \right] d\,z\,d\,y\,d\,x \tag{11}$$

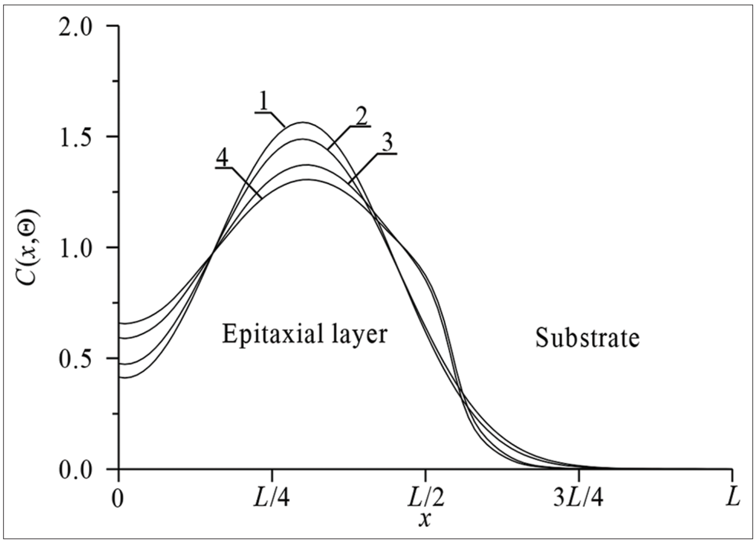

Figure 3: Distributions of concentration of implanted dopant in heterostructure from Figure 1 in direction, which is perpendicular to the interface between epitaxial layer substrate. Curves 1 and 3 corresponds to annealing time $\Theta=0.0048(L_x^2+L_y^2+L_z^2)/D_0$. Curves 2 and 4 correspond to annealing time $\Theta=0.0057(L_x^2+L_y^2+L_z^2)/D_0$. Curves 1 and 2 correspond to the homogenous sample. Curves 3 and 4 correspond to heterostructure under condition, when the value of dopant diffusion coefficient in epitaxial layer is larger than the value of dopant diffusion coefficient in the substrate

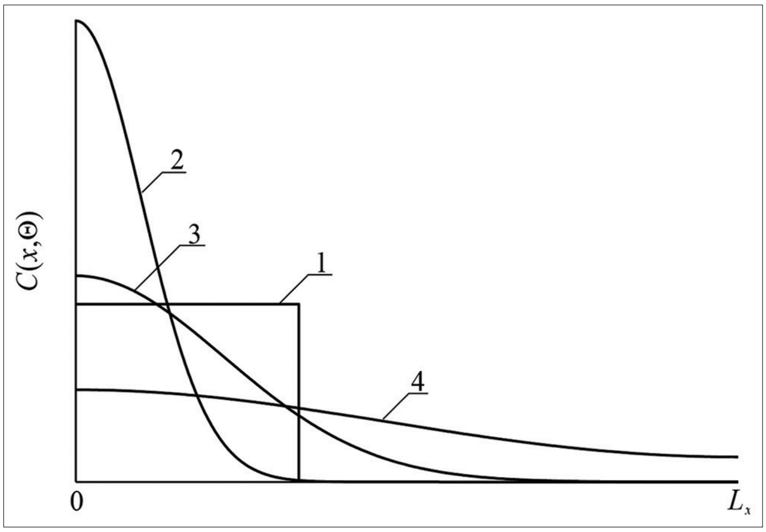

Figure 4: Spatial distributions of dopant in heterostructure after dopant infusion. Curve 1 is the idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time. Increasing of number of the curve corresponds to increasing of annealing time

where Ψ (x,y,z) is the approximation function. Dependences of optimal values of annealing time on parameters are presented on Figures 6 and 7 for diffusion and ion types of doping, respectively. It should be noted that it is necessary to anneal radiation defects after ion implantation. One could find the spreading of concentration of distribution of dopant during this annealing. In the ideal case distribution of dopant achieves appropriate interfaces between materials of heterostructure during annealing of radiation defects. If dopant did not achieve any interfaces during annealing of radiation defects, it is practicably to additionally anneal the dopant. In this situation, the optimal value of additional annealing time of implanted dopant is smaller than annealing time of infused dopant.

Farther, we analyzed the influence of relaxation of mechanical stress on the distribution of dopant in doped areas of the heterostructure. Under the following conditions $\varepsilon_0<0$, one can find compression of distribution of concentration of dopant near the interface between materials of the heterostructure.

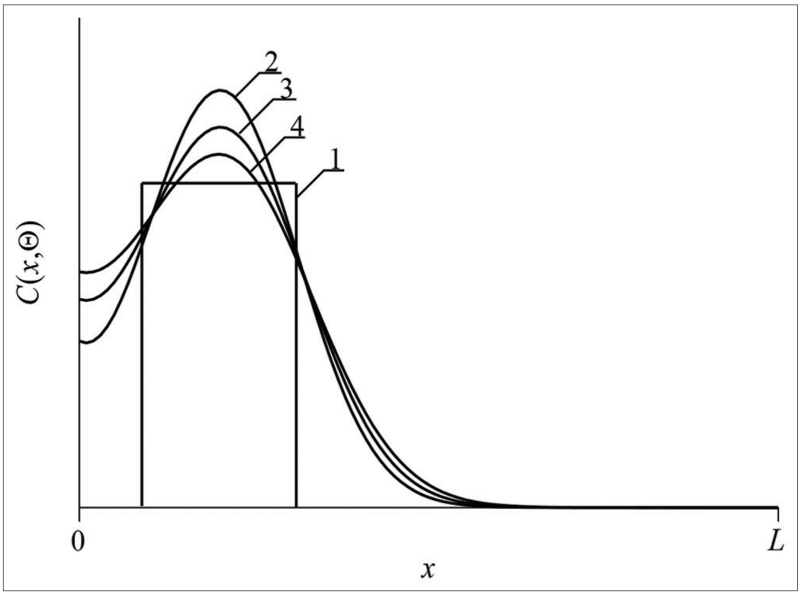

Figure 5: Spatial distributions of dopant in heterostructure after ion implantation. Curve 1 is idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time

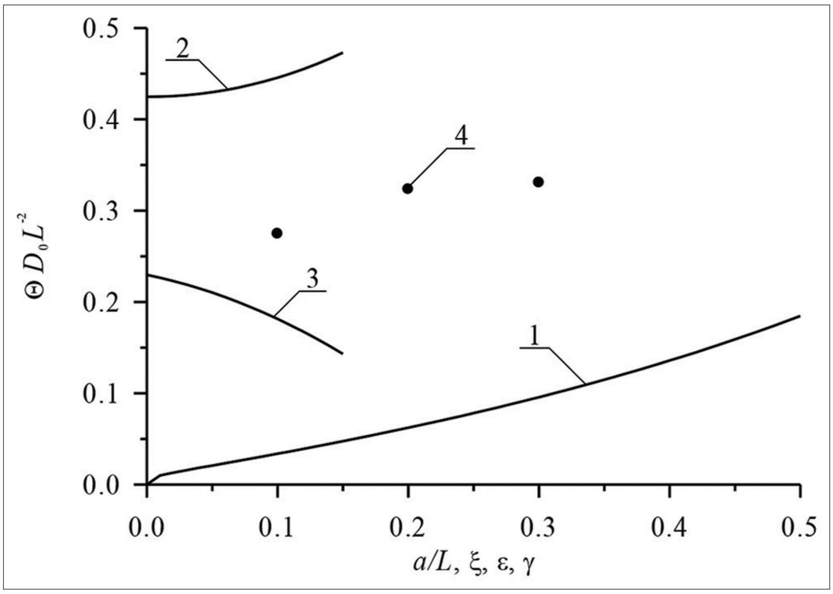

Figure 6: Dependences of dimensionless optimal annealing time for doping by diffusion, which have been obtained by minimization of mean-squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation a/L and $\xi=\gamma=0$ for equal to each other values of the dopant diffusion coefficient in all parts of the heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for $a/L=1/2$ and $\xi=\gamma=0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter ξ for $a/L=1/2$ and $\varepsilon=\gamma=0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for $a/L=1/2$ and $\varepsilon=\xi=0$

Contrary (at ε_0>0) one can find the spreading of distribution of concentration of dopant in this area. This changing of distribution of concentration of dopant could be at least partially compensated using laser annealing.[37] This type of annealing gives us the possibility to accelerate the diffusion of dopant and another processes in the annealed area due to inhomogeneous distribution of temperature and Arrhenius law. Accounting relaxation of mismatch-induced stress in heterostructure could lead to the changing of optimal values of annealing time. At the same time, modification of porosity gives us the possibility to decrease the value of mechanical stress. On the one hand, mismatch-induced stress could be used to increase the density of elements of integrated circuits. On the other hand, it could lead to generation dislocations of the discrepancy. Figures 8 and 9 show distributions of the concentration of vacancies in porous materials and component of the displacement vector, which is perpendicular to the interface between layers of heterostructure.

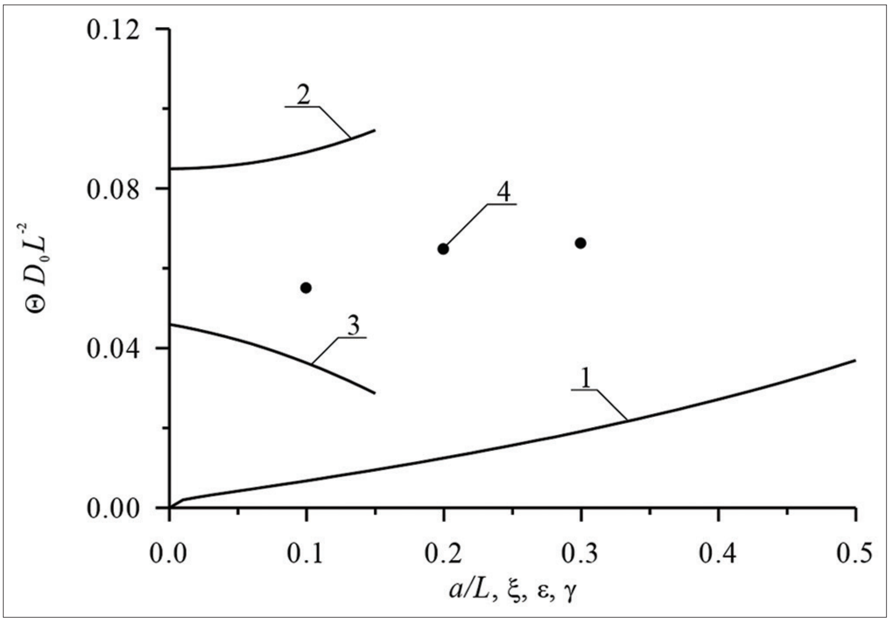

Figure 7: Dependences of dimensionless optimal annealing time for doping by ion implantation, which have been obtained by minimization of mean-squared error, on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation a/L and $\xi=\gamma=0$ for equal to each other values of the dopant diffusion coefficient in all parts of the heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for $a/L=1/2$ and $\xi=\gamma=0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter ξ for $a/L=1/2$ and $\varepsilon=\gamma=0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for $a/L=1/2$ and $\varepsilon=\xi=0$

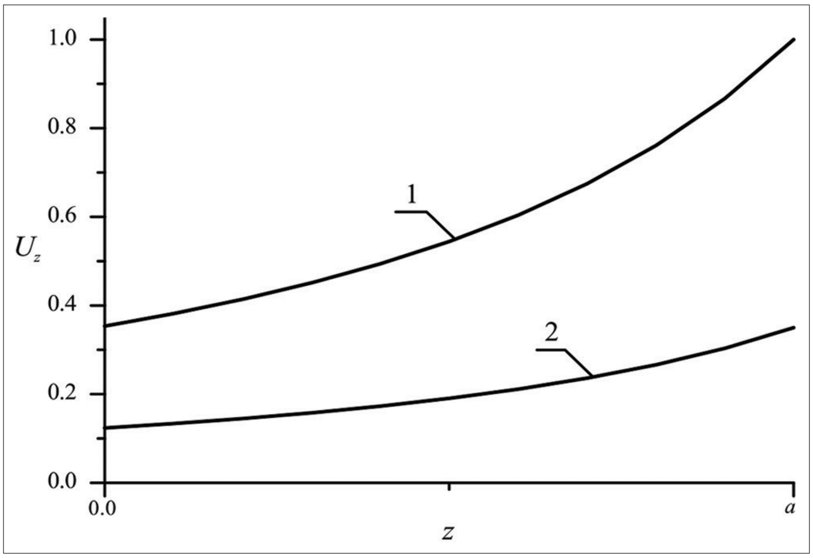

Figure 8: Normalized dependences of component u_z of displacement vector on coordinate z for nonporous (curve 1) and porous (curve 2) epitaxial layers

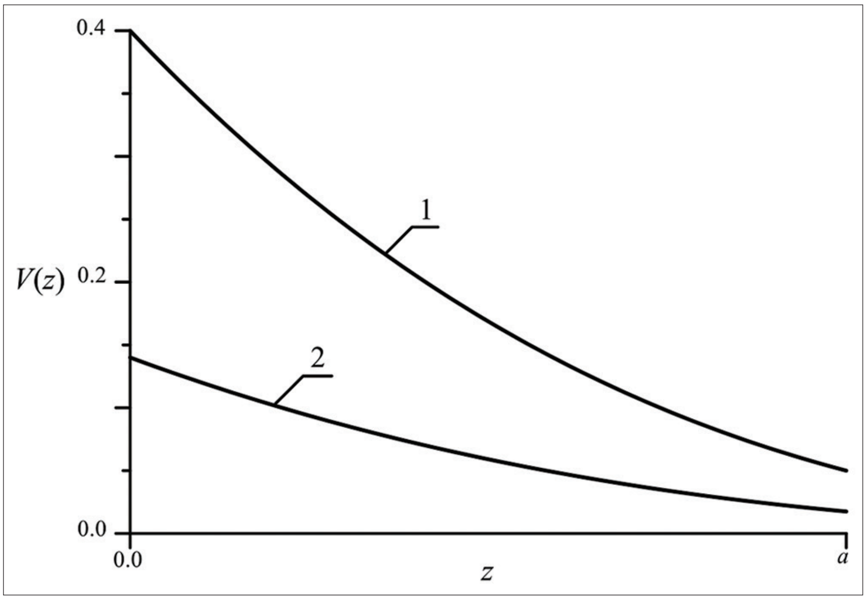

Figure 9: Normalized dependences of vacancy concentrations on coordinate z in unstressed (curve 1) and stressed (curve 2) epitaxial layers

CONCLUSION

In this paper, we model redistribution of infused and implanted dopants with account relaxation mismatch-induced stress during manufacturing field-effect heterotransistors framework two-level current-mode logic gates in a multiplexer. We formulate recommendations for the optimization of annealing to decrease dimensions of transistors and to increase their density. We formulate recommendations to decrease mismatch-induced stress. Analytical approach to model diffusion and ion types of doping with account concurrent changing of parameters in space and time has been introduced. At the same time, the approach gives us the possibility to take into account the nonlinearity of considered processes.

REFERENCES

1. Lachin VI, Savelov NS. Electronics. Rostov-on-Don: Phoenix; 2001.
2. Polishscuk A. Modern models of integrated operating amplifier. Mod Electron 2004;12:8-11.
3. Volovich G. Modern chips UM3Ch class D manufactured by firm MPS. Mod Electron 2006;2:10-7.
4. Kerentsev A, Lanin V. Constructive-technological features of MOSFET-transistors. Power Electron 2008;1:34.
5. Ageev AO, Belyaev AE, Boltovets NS, Ivanov VN, Konakova RV, Kudrik YY, *et al*. Sachenko. Semiconductors 2009;43:897-903.
6. Tsai JH, Chiu SY, Lour WS, Guo DF. High-performance InGaP/GaAs pnp δ-doped heterojunction bipolar transistor. Semiconductors 2009;43:939-42.
7. Alexandrov OV, Zakhar'AO ZZ, Sobolev NA, Shek EI, Makoviychuk MM, Parshin EO. Formation of donor centers after annealing of dysprosium and holmium implanted silicon. Semiconductors 1998;32:1029-32.
8. Ermolovich IB, Milenin VV, Red'RA RR, Red'SM RR. Specific features of recombination processes in CdTe films produced in different temperature conditions of growth and subsequent annealing. Semiconductors 2009;43:1016-20.
9. Sinsermsuksakul P, Hartman K, Kim SB, Heo J, Sun L, Park HH, *et al*. Enhancing the efficiency of SnS solar cells via band-offset engineering with a zinc oxysulfide buffer layer. Appl Phys Lett 2013;102:53901-5.
10. Reynolds JG, Reynolds CL, Mohanta A Jr., Muth JF, Rowe JE, Everitt HO. Shallow acceptor complexes in p-type ZnO. Appl Phys Lett 2013;102:152114-8.
11. Volokobinskaya NI, Komarov IN, Matyukhina TV, Reshetnikov VI, Rush AA, Falina IV, *et al*. Investigation of technological processes of manufacturing of the bipolar power highvoltage transistors with a grid of inclusions in the collector region. Semiconductors 2001;35:1013-7.
12. Pankratov EL, Bulaeva EA. Optimal criteria to estimate temporal characteristics of diffusion process in a media with inhomogenous and nonstationary parameters. Rev Theor Sci 2013;1:58-82.
13. Kukushkin SA, Osipov AV, Romanychev AI. Epitaxial growth of zinc oxide by the method of atomic layer deposition on SiC/Si substrates. Phys Solid State 2016;58:1448-52.
14. Trukhanov EM, Kolesnikov AV, Loshkarev ID. Long++range stresses generated by misfit dislocations in epitaxial films. Russ Microelectron 2015;44:552-8.

15. Pankratov EL, Bulaeva EA. About some ways to decrease quantity of defects in materials for solid state electronic devices and diagnostics of their realization. Rev Theor Sci 2015;3:365-98.
16. Ong KK, Pey KL, Lee PS, Wee AT, Wang XC, Chong YF. Dopant distribution in the recrystallization transient at the maximum melt depth induced by laser annealing. Appl Phys Lett 2006;89:172111-4.
17. Wang HT, Tan LS, Chor EF. Pulsed laser annealing of Be-implanted GaN. J Appl Phys 2006;98:94901-5.
18. Bykov YV, Yeremeev AG, Zharova NA, Plotnikov IV, Rybakov KI, Drozdov MN, *et al*. Diffusion processes in semiconductor structures during microwave annealing. Radiophys Quantum Electron 2003;43:836-43.
19. Jang I, Kim J, Kim S. Accurate delay models of CMOS CML circuits for design optimization. Analog Integr Circuits Sig Process 2015;82:297-307.
20. Zhang YW, Bower AF. Numerical simulations of island formation in a coherent strained epitaxial thin film system. J Mech Phys Solids 1999;47:2273-97.
21. Landau LD, Lefshits EM. Theory of elasticity. In: Theoretical Physics. Vol. 7. Moscow: Physmatlit; 2001.
22. Kitayama M, Narushima T, Carter WC, Cannon RM, Glaeser AM. The Wulff shape of alumina: I, modeling the kinetics of morphological evolution. J Am Ceram Soc 2000;83:2531-61. Kitayama M, Narushima T, Glaeser AM. The Wulff shape of alumina: II, experimental measurements of pore shape evolution rates. J Am Ceram Soc 2000;83:2572-83.
23. Cheremskoy PG, Slesov VV, Betekhtin VI. Pore in Solid Bodies. Moscow: Energoatomizdat; 1990.
24. Gotra ZY. Technology of Microelectronic Devices. Moscow: Radio and Communication; 1991.
25. Fahey PM, Griffin PB, Plummer JD. Point defects and dopant diffusion in silicon. Rev Mod Phys 1989;61:289-388.
26. Vinetskiy VL, Kholodar GA. Radiative Physics of Semiconductors. Kiev: Naukova Dumka; 1979.
27. Mynbaeva MG, Mokhov EN, Lavrent'AA LL, Mynbaev KD. High-temperature diffusion doping of porous silicon carbide. Tech Phys Lett 2008;34:13.
28. Sokolov YD. About determination of dynamical forces in mine's hoisting ropes. Appl Mech 1955;1:23-35.
29. Pankratov EL. Dopant diffusion dynamics and optimal diffusion time as influenced by diffusion-coefficient nonuniformity. Russ Microelectron 2007;36:33-9.
30. Pankratov EL. Redistribution of dopant during annealing of radiative defects in a multilayer structure by laser scans for production an implanted-junction rectifiers. Int J Nanosci 2008;7:187-97.
31. Pankratov EL, Bulaeva EA. Doping of materials during manufacture p-n-junctions and bipolar transistors. Analytical approaches to model technological approaches and ways of optimization of distributions of dopants. Rev Theor Sci 2013;1:58-82.
32. Pankratov EL, Bulaeva EA. Decreasing of quantity of radiation defects in an implanted-junction rectifiers by using overlayers. Int J Micro Nano Scale Transp 2012;3:119-30.
33. Pankratov EL, Bulaeva EA. Optimization of manufacturing of emitter-coupled logic to decrease surface of chip. Int J Mod Phys B 2015;29:1550023.
34. Pankratov EL. On approach to optimize manufacturing of bipolar heterotransistors framework circuit of an operational amplifier to increase their integration rate. Influence mismatch-induced stress. J Comp Theor Nanosci 2017;14:4885-99.
35. Pankratov EL, Bulaeva EA. An approach to increase the integration rate of planar drift heterobipolar transistors. Mater Sci Semicond Process 2015;34:260-8.
36. Pankratov EL, Bulaeva EA. An approach to manufacture of bipolar transistors in thin film structures. Int J Micro Nano Scale Transp 2014;4:17-31.
37. Pankratov EL, Bulaeva EA. An analytical approach for analysis and optimization of formation of field-effect heterotransistors. Multidiscip Mod Mater Struct 2016;12:578.