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A Monte Carlo Study of Empirical Performance of Threshold Autoregressive Models on Nonlinear Non-Stationary

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ABSTRACT

One of the major importance of modeling in time series is to forecast future values of that series which requires the use of appropriate method to fit the time series data that are dependent on the nature of the data. However, real-life data are mostly non-stationary and nonlinear. This will be a problem when a model is in appropriately applied to data that does not fits in, the result of the outcome will be inaccurate and this will not give the clear picture of what the data entails in the future events. In this study, the performances of the smooth transition autoregressive (STAR) and the self-exciting threshold autoregressive (SETAR) models of different orders and regimens were compared on different forms of nonlinear cases of autoregressive under violation of stationarity assumption. Simulated data with features of nonlinearity and non-stationarity were used to compare the performance of the models. The relative performances of the models were examined with a view to identify the best models at orders 1, 2, and 3, and regimen 2 when fitted to linear, trigonometric, exponential, and polynomial autoregressive functions. It was concluded that the SETAR (2, 1) is the best model followed by SETAR (2, 2) to fit linear data, whereas the SETAR (2, 2) and STAR (2, 3) are considered to be the best for an exponential and SETAR (2, 2) and STAR (2, 2) for a polynomial data.

Key words: SETAR, STAR, Simulation, MSE, MAPE, AIC

INTRODUCTION

A time series forecasting is the use of model to predict the future values based on the past values. Predictions were made when the actual outcome of event(s) may not be known until some future time (Akeyede *et al*, 2015).[1] The goal of time series is to forecast and identify meaningful characteristics in data that can be used in making statements about the future outcomes. Time series is generally classified into stationary and non-stationary. A stationary time series has its statistical properties such as the mean, variance, autocovariance, and autocorrelation are all constant over time. Since its characteristics are constant, then a stationarized time series can be easily predicted because all its statistical properties that were constant in the past will for likely be constant at future. On the other hand, a non-stationary time series is the one whose statistical properties change over time. Moreover, this need to be further converted into stationary data because using non-stationary time series, especially in financial models produces unreliable result and leads to poor understanding and forecasting (Akeyede *et al,* 2016).[2]

The proposal of nonlinear models is one of the most important methods in time series analysis, which has a wide potential for predicting various phenomena, including physical sciences, engineering, and economics, by studying the characteristics of random disturbances to arrive at accurate predictions. The most important method of dealing with nonlinear time series data is the threshold autoregressive model (TAR) which was developed by Tong (1978)[3] and has received a great attention in the nonlinear time series. Literature has been widely used in various fields such as econometrics and finance among others (see Tong (1978)[3] and Akeyede *et al*, 2016[4]).

TAR is one of the nonlinear models applicable nowadays. It was developed by Tong (1978)[3] and discussed in detail by Tong and Lim (1980)[5] and later by Tong (1983).[6] TARs are often difficult

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to model and this is due to the fact that it lacks a suitable modeling procedure and inability to identify the threshold variables and estimate the threshold values (Aydin, 2015 and Clement and Smith, 1997 and 2001).[7-9]

Therefore, this study considers some nonlinear threshold models, namely smooth transition autoregressive (STAR) models and the self-exciting threshold autoregressive (SETAR) models of different orders. Their forecast performances were examined on different forms of nonlinear classes of autoregressive under violation of stationarity assumption through Monte Carlo simulation.

Self-Exiting Threshold Autoregressive Models

The SETAR model is arguably the most widely used scalar nonlinear time series model. It is an extension of the piecewise linear regression model (or segmented linear regression model) with structural changes occurring in the threshold space. In the time series literature, the SETAR model was proposed by Tong (1978)[10] and has been widely used for the publication of Tong and Lim (1980)[5], Keenan (1985)[10], Peel (1998).[11] Lundbergh *et al* (2002)[12] and Jorge *et al* (2005).[13] The simple case of the two-regimen SETAR model is indicated below.

$$X_t = \begin{cases} \varnothing_0 + \sum_{i=1}^{P} \varnothing_i X_{t-i} + \sigma_i \epsilon_t \; if \; X_{t-d} \le r \\ \theta_0 + \sum_{i=1}^{p} \theta i X_{t-i} + \sigma_2 \epsilon_{t,} \; if \; X_{t-d} > r \end{cases} \quad (3.10)$$

where ϵ_t is a sequence of independently and identically distributed random variables with mean zero and unit variance, $\emptyset_i$ and θ_i are real-valued parameters such that $\emptyset_i = \theta_i$ for some *i*, *d* is a positive integer denoting the delay, and *r* is the threshold value. Often, this further assumes that ϵ_t follows N (0,1). We use the same order *p* for both regimens. This is purely for simplicity as different orders can easily be used (Akeyede *et al,* 2015).[14]

Smooth Transition Autoregressive (STAR) Model

Another class of nonlinear time series models is smooth transition autoregressive (STAR) models. The STAR model is similar to the SETAR model. The main difference between these two models is the mechanism governing the transition between regimens. A two-regimen model will be considered here. The concept can be extended to the case with more than two regimens (Fırat, 2007, Guidolin,2009 and Ismail, 2006).[15-17]

A criticism of the SETAR model is that its conditional mean equation is not continuous. The thresholds (r_j) are the discontinuity points of the conditional mean function (μ_t) (Samson *et al*, 2015, Sarantis, 1999, and Tayyab, 2012).[18-20] In response to this criticism, the STAR models have been proposed. Consider time series X_t that follows a two-regimen STAR (p) model of the form

$$X_t = c_0 + \sum_{i=1}^{p} \varnothing_{0.i} X_{t-i} + F\left(\frac{X_{t-d} - \Delta}{s}\right) \left(c_1 \sum_{i=1}^{p} \varnothing_{1.i} X_{t-i}\right) + e_t \quad (3.19)$$

Where *d* is the delay parameter, Δ and s are parameters representing the location and scale of model transition, and $F(\cdot)$ is a smooth transition function. In practice, $F(\cdot)$ often assumes one of three forms, namely logistic, exponential, or a cumulative distribution function. The conditional mean of a STAR model is a weighted linear combination between the following two equations:

$$\mu_{1t} = c_0 + \sum_{i=1}^{p} \varnothing_{0.i} X_{t-i} \quad (3.20a)$$

$$\mu_{2t} = (c_0 + c_1) + \sum_{i=1}^{p} (\varnothing_{0.i} + \varnothing_{1.t}) X_{t-1} \quad (3.20b)$$

The weights are determined in a continuous manner by $F\left(\frac{Y_{t-d} - \Delta}{s}\right)$. The prior two equations above also determine the properties of a STAR model. For instance, a prerequisite for the stationary of a STAR model is that all zeros of both AR polynomials are outside the unit circle. An advantage of the STAR model over the SETAR model is that the conditional mean function is differentiable. However, experience shows that the transition parameters Δ and s of a STAR model are hard to estimate. In particular, most empirical studies show that standard errors of the estimates of Δ and s are often quite large, resulting in t ratios of about 1.0. This uncertainty leads to various complications in interpreting an estimated STAR model (Terasvirta, 2005, Tsay 1986, Umer et al, 2018 and Zhou, 2010).[21-24]

MATERIALS AND METHODS

Simulation studies were conducted to investigate the performance of autoregressive, SETA and STAR models for fitting different general classes of nonlinear autoregressive time series earlier stated. The effect of sample size and the non-stationarity of the models were examined on each of the general linear and nonlinear data simulated. Each model is subjected to 1000 replication simulation at different sizes for non-stationary data structures.

Criteria for Assessment of the Study

The goodness of fit for each model was assessed using common series, mean square error (MSE), mean absolute percentage error (MAPE), and the Akaike information criteria (AIC). The model with the lowest criteria is the best among the models for the simulated data.

Mean squared error

The mean squared error (MSE) of an estimator measures the average of the squares of the "errors," that is, the difference between the estimator and what is estimated.

If $\hat{Y}$ is a vector of estimated values, and Y is the vector of the true values, then the (estimated) MSE

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{Y}_i - Y_i)^2$$

Mean absolute percentage error (MAPE)

The mean absolute percentage error (MAPE), also known as mean absolute percentage deviation (MAPD), is a measure of accuracy of a method for constructing fitted time series values in statistics, specifically in model fitting. It usually expresses accuracy as a percentage, and is defined by the formula:

$$M = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{A_t - F_t}{A_t}\right|$$

where A_t is the actual value and F_t is the forecast value. The difference between A_t and F_t is divided by the actual value A_t again. The absolute value in this calculation is summed for every fitted or forecasted point in time and divided again by the number of fitted points *n*. Multiplying by 100 makes it a percentage error.

Akaike information criteria

Supposing we have a statistical model of some data, let k be the number of estimated parameters included in the model and *n*, sample size; then, the AIC value of the model is

$$AIC = nln\left(\hat{\sigma}^2\right) + 2k$$

where

$$\hat{\sigma}^2 = \frac{Residual\, sum\, of\, squares}{n}.$$

The *ln* (*likelihood*) of the model given in the data, is readily available in statistical output and reflects the overall fit of the model.

Selection Rule

We compute the MSE, MAPE, AIC, and MAPE for *n* = 20, 40, 60, 80, 150, 100, 120, 140, 160, 180, and 200 for each case model, and select the model that has the minimum criteria values as the best. Note that three orders of autoregressive were considered in each case and situation.

Models Selected for Simulation

Data are generated from several linear and nonlinear second orders of general classes of autoregressive models as given below:

Model 1: AR (2): $X_{ti} = 0.7\, X_{ti-1} - 0.6\, X_{ti-2} + e_t$

Model 2: TR (2): $X_{ti} = 0.7 \sin(X_{ti-1}) - 0.6 \cos(X_{ti-2}) + e_t$

Model 3: EX (2): $X_{ti} = 0.7\, X_{ti-2} - \exp(0.6X_{ti-2}) + e_t$

Model 4: PL (2): $X_{ti} = 0.7 X_{ti-1}^2 - 0.6X_{ti-2} + e_t$

X_{ti}~N(2,4) and e_{ti}~N(1,2)

t = 1, 2,…, 20, 40, 60, 80, 100, 120, 140, 160, 180 and 200. *i* = 1, 2,…, 1000

The model 1, 2, 3, and 4 are linear, trigonometry, exponential, and polynomial autoregressive functions, respectively, with coefficients of X_{t-1} being 0.7, X_{t-2} being −0.6. Simulation studies were conducted to investigate the performance of SETAR and STAR models for fitting different general classes of linear and nonlinear autoregressive time series stated above. The effect of sample size and the non-stationarity of the models were also examined on the models.

Table 1: Mean square error of self-exciting threshold autoregressive and smooth transition autoregressive models fitted on AR (2): Xti=0.7X_{ti-1}−0.6X_{ti-2}+e_t across the sample sizes

Sample size	SETAR (2,1)	SETAR (2,2)	SETAR (2,3)	STAR (2,1)	STAR (2,2)	STAR (2,3)
20	3.4916	3.3893	4.8638	4.1091	4.4153	3.5648
40	3.9660	3.7852	5.8296	4.1983	4.1402	4.1849
60	4.0522	4.0157	5.3391	4.9072	4.9424	4.8346
80	3.9358	4.1073	5.0705	4.6135	4.6693	4.6003
100	3.9329	4.0630	5.1909	4.1576	4.2277	4.1639
120	3.7256	3.8715	5.0806	4.0375	4.0628	4.1595
140	3.7219	3.7614	4.5859	3.9492	3.9252	4.0130
160	3.7069	3.6009	4.0380	3.9196	3.9352	3.9730
180	3.7286	3.5142	3.9421	3.9159	3.9050	3.9380
200	3.5581	3.1785	3.9928	3.7854	3.7638	3.7683

SETAR: Self-exciting threshold autoregressive, STAR: Smooth transition autoregressive

Table 2: Mean square error of self-exciting threshold autoregressive and smooth transition autoregressive models fitted on AR (2): X_{ti}=0.7X_{ti}−1-0.6X_{ti}-2+e_t across the sample sizes

Sample size	SETAR (2,1)	SETAR (2,2)	SETAR (2,3)	STAR (2,1)	STAR (2,2)	STAR (2,3)
20	0.1191	0.1198	0.1196	0.1195	0.1197	0.1192
40	0.1187	0.1198	0.1196	0.1195	0.1198	0.1190
60	0.1188	0.1199	0.1193	0.1195	0.1199	0.1190
80	0.1186	0.1196	0.1190	0.1192	0.1199	0.1187
100	0.1187	0.1195	0.1193	0.1193	0.1197	0.1189
120	0.1179	0.1196	0.1194	0.1192	0.1195	0.1184
140	0.1184	0.1195	0.1192	0.1193	0.1196	0.1182
160	0.1180	0.1195	0.1189	0.1188	0.1194	0.1181
180	0.1178	0.1194	0.1186	0.1183	0.1193	0.1180
200	0.1179	0.1195	0.1187	0.1187	0.1193	0.1182

SETAR: Self-exciting threshold autoregressive, STAR: Smooth transition autoregressive

Table 3: Akaike information criteria of self-exciting threshold autoregressive and smooth transition autoregressive fitted on AR (2): Xti=0.7 X_{ti-1}-0.6 X_{ti-2}+e_t models across the sample sizes

Sample size	SETAR (2,1)	SETAR (2,2)	SETAR (2,3)	STAR (2,1)	STAR (2,2)	STAR (2,3)
20	39.0071	31.4197	38.8472	34.2640	35.7014	30.3998
40	63.1104	64.2440	99.3844	69.3870	67.8300	65.2588
60	101.9361	102.0535	145.4402	101.4427	101.8713	102.5476
80	129.4853	112.8418	213.8843	128.3188	129.2798	130.0890
100	150.9370	148.1906	247.2784	148.4934	150.1648	150.6457
120	171.8259	170.4354	254.7929	173.4744	174.2252	179.0462
140	197.9930	197.4720	255.5188	198.2904	197.4382	202.5348
160	223.6301	225.6387	268.5576	224.5566	225.1940	228.7234
180	250.8862	247.9722	275.4175	251.7081	251.2052	254.7187
200	267.8437	264.5026	288.6473	272.2327	271.0831	273.3267

SETAR: Self-exciting threshold autoregressive, STAR: Smooth transition autoregressive

DATA ANALYSIS

The performances of the fitted model on the basis of the three criteria were displayed in Tables 1-12 as follows

Table 1 shows the relative performance of the fitted models at different levels of sample size based on MSE criteria. It was observed that SETAR (2,1) and SETAR (2,2) models have the best performance model from the various sample sizes to fit linear form of autoregressive models that can be used to forecast for future values at different steps ahead. SETAR (2,3) performs far worse than the other models from the beginning but as the sample size increases, the trend tends to be a little stable.

Table 2 shows the relative performance of the fitted models at different levels of sample size based on MAPE criteria. SETAR (2,1) followed by STAR (2,3) models at different orders shows the best performance from the various sample sizes to linear form of autoregressive models

Table 3 shows the relative performance of the fitted models at different levels of sample size based on AIC criteria. It was observed that SETAR (2, 1) and SETAR (2, 2) of the models leading the other models at different orders show the best performance model from the various sample sizes to linear form of autoregressive models that can be used to forecast for future values at different steps ahead. SETAR (2, 3) performs far worse than the other models as the trends increase higher than the other models but later, the trend reduces and rate close to the other models.

Table 4 shows the relative performance of the fitted models at different levels of sample size based on

Table 4: Mean square error of self-exciting threshold autoregressive and smooth transition autoregressive models fitted on TR (2): X_{ti}=0.7 sin (X_{ti-1})−0.6 cos (X_{ti-2})+e_t across the sample sizes

Sample size	SETAR (2,1)	SETAR (2,2)	SETAR (2,3)	STAR (2,1)	STAR (2,2)	STAR (2,3)
20	3.1317	3.9799	5.1181	4.9133	6.2620	2.9152
40	3.8272	4.0402	9.2536	4.6781	5.2848	4.3708
60	4.7649	5.2886	9.2200	5.0171	6.0036	5.0686
80	4.3264	5.0239	10.5929	5.0076	5.5915	5.1949
100	4.0247	4.4264	9.2359	4.0140	4.9518	4.4327
120	4.1275	4.3633	10.8565	4.1048	4.8799	4.5714
140	3.9586	4.1253	14.9776	3.9784	4.6057	4.2851
160	3.9487	4.0205	18.3391	3.8492	4.6082	4.3944
180	3.9404	3.9853	12.3866	3.6468	4.5457	3.9673
200	3.8162	3.9065	15.6097	3.5532	4.3554	4.4240

SETAR: Self-exciting threshold autoregressive, STAR: Smooth transition autoregressive

Table 5: Mean absolute percentage error of self-exciting threshold autoregressive and smooth transition autoregressive models fitted on TR (2): X_{ti}=0.7 sin (X_{ti-1})−0.6 cos (X_{ti-2})+e_t across the sample sizes

Sample size	SETAR (2,1)	SETAR (2,2)	SETAR (2,3)	STAR (2,1)	STAR (2,2)	STAR (2,3)
20	1.8096	1.2232	0.3113	2.0948	1.7716	0.3272
40	1.6180	1.1999	0.6146	1.7030	1.3015	0.4018
60	2.1354	1.4137	0.7799	2.2351	1.5232	0.2263
80	1.7134	1.1274	0.2609	2.1014	1.5958	0.1241
100	1.5243	1.1416	0.2443	1.4898	1.2845	0.1790
120	1.7059	2.3918	0.9492	1.3879	2.2063	0.2119
140	1.6949	2.6559	0.4590	1.7847	2.8363	0.5093
160	1.6691	3.0106	0.3566	1.6834	3.9757	0.6079
180	1.6578	3.1846	0.4930	1.6911	3.2817	0.1740
200	1.5990	2.0819	0.9717	1.6339	3.5406	0.1868

SETAR: Self-exciting threshold autoregressive, STAR: Smooth transition autoregressive

Table 6: Akaike information criteria of self-exciting threshold autoregressive and smooth transition autoregressive models fitted on TR (2): X_{ti}=0.7 sin (X_{ti-1})−0.6 cos (X_{ti-2})+e_t across the sample sizes

Sample size	SETAR (2,1)	SETAR (2,2)	SETAR (2,3)	STAR (2,1)	STAR (2,2)	STAR (2,3)
20	36.8313	41.6249	29.0102	37.8389	42.6899	29.3989
40	67.6850	69.85207	132.3078	67.7157	72.5929	66.9982
60	107.6771	113.9337	216.4913	108.4712	113.5413	105.3834
80	131.1796	143.1360	247.8224	138.4694	143.7004	139.8138
100	153.2460	162.7592	333.0052	154.9776	165.9744	156.8997
120	184.1218	190.7881	363.8278	185.4591	196.2145	190.3778
140	206.6239	212.3983	399.2453	209.3212	219.8211	211.7200
160	233.7404	236.6254	402.7533	231.6594	250.4544	244.8519
180	260.8318	262.8701	479.2933	258.8910	278.5537	256.0564
200	281.8490	286.5255	523.4700	279.5675	300.2820	305.4102

SETAR: Self-exciting threshold autoregressive, STAR: Smooth transition autoregressive

MSE criteria. It was observed that SETAR (2, 1) and STAR (2, 1) of the models leading the other models at smaller and larger sample sizes, respectively, to fit the trigonometric form of autoregressive and therefore can be used to forecast for future values at different steps ahead for the respective sample sizes. SETAR (2, 3) performs far worse than the other models as the trends increases.

Table 5 shows the relative performance of the fitted models at different levels of sample size based on MAPE criteria. It was observed that SETAR (2, 3) followed by STAR (2, 3) shows the best performance from the various sample sizes to trigonometric form of autoregressive models.

Table 6 shows the relative performance of the fitted models at different levels of sample size

Table 7: Mean square error of self-exciting threshold autoregressive and smooth transition autoregressive models fitted on EX (2): X_{ti}=0.7 X_{ti-2})−exp (0.6X_{ti-2})+e_t across the sample sizes

Sample size	SETAR (2,1)	SETAR (2,2)	SETAR (2,3)	STAR (2,1)	STAR (2,2)	STAR (2,3)
20	3.02894	2.8190	3.6895	3.3292	4.2523	3.8184
40	3.6768	3.1874	5.9413	4.3560	4.1094	3.5792
60	4.3777	4.1330	6.5362	4.7981	4.8647	4.2307
80	4.3474	4.1861	6.5450	4.6497	4.6758	4.3435
100	4.1241	3.9152	6.2770	4.5205	4.2475	4.1224
120	4.0419	3.9094	5.4367	4.1803	4.1205	4.0421
140	3.9211	3.8008	5.7090	4.1060	4.0306	3.9237
160	3.9863	4.1000	4.6101	4.0739	4.3352	3.9861
180	4.0032	4.1030	4.7183	4.1407	4.1641	3.8738
200	3.8448	3.8091	4.8949	4.1562	3.9389	3.7661

SETAR: Self-exciting threshold autoregressive, STAR: Smooth transition autoregressive

Table 8: Mean absolute percentage error of self-exciting threshold autoregressive and smooth transition autoregressive models fitted on EX (2): X_{ti}=0.7 X_{ti-2}−exp (0.6X_{ti-2})+e_t across the sample sizes

Sample size	SETAR (2,1)	SETAR (2,2)	SETAR (2,3)	STAR (2,1)	STAR (2,2)	STAR (2,3)
20	1.3342	0.4047	0.1393	2.2055	1.3890	0.0856
40	1.6161	0.9140	0.1453	1.5373	1.0737	0.0353
60	1.8865	1.1834	0.1516	2.0111	1.3406	0.0166
80	1.5033	1.1021	0.0433	1.4952	1.4610	0.0251
100	1.7352	1.0882	0.0691	1.7313	1.2953	0.0164
120	1.6202	1.1602	0.0795	1.6204	1.1695	0.0255
140	1.5126	1.0245	0.1535	1.5145	0.9938	0.0239
160	2.7140	0.9093	0.1059	2.7156	0.9457	0.0128
180	2.5708	1.2089	0.1491	2.7088	1.0777	0.0205
200	2.9151	1.1651	0.2868	3.0500	1.2230	0.0284

SETAR: Self-exciting threshold autoregressive, STAR: Smooth transition autoregressive

Table 9: Akaike information criteria of self-exciting threshold autoregressive and smooth transition autoregressive models fitted on EX (2): X_{ti}=0.7 X_{ti-2}−*exp* (0.6X_{ti-2})+e_t across the Sample Sizes

Sample size	SETAR (2,1)	SETAR (2,2)	SETAR (2,3)	STAR (2,1)	STAR (2,2)	STAR (2,3)
20	28.1502	34.7274	33.7870	37.4488	34.9490	32.0544
40	66.0818	60.3679	99.7018	66.8613	62.5307	66.8624
60	102.5909	99.1399	122.6586	108.6190	100.9202	102.0925
80	131.5660	128.5413	181.0831	133.4939	129.3918	130.9438
100	155.6841	150.4854	225.2943	157.6435	150.6339	152.6946
120	181.6056	177.6071	298.4070	183.6111	175.9158	179.6469
140	205.2917	200.9311	334.8540	207.3851	201.1480	192.3197
160	235.2588	231.6578	376.7380	237.2519	240.6836	228.7670
180	263.6756	258.8677	397.9810	259.7636	262.7697	254.1874
200	283.3450	271.4776	400.4040	281.2083	280.1821	266.0767

SETAR: Self-exciting threshold autoregressive, STAR: Smooth transition autoregressive

Table 10: Mean square error of self-exciting threshold autoregressive and smooth transition autoregressive models fitted on PL (2): Xti= $0.7X_{ti-1}^2 - 0.6X_{ti-2} + e_t$ across the sample sizes

Sample size	SETAR (2,1)	SETAR (2,2)	SETAR (2,3)	STAR (2,1)	STAR (2,2)	STAR (2,3)
20	853.4312	587.3325	757.1245	645.3714	395.0563	745.3714
40	793.5645	477.3434	703.3665	585.3477	337.4387	725.3426
60	777.1982	398.0101	694.7014	503.1258	320.3332	703.1235
80	741.2009	387.4829	659.2508	567.5661	304.9870	687.5608
100	721.0786	218.8977	646.2354	517.8725	299.4432	657.7815
120	693.3134	197.4535	612.0578	414.1761	282.8970	624.1766
140	656.0975	193.7908	585.3008	460.1515	270.0070	616.1530
160	614.7413	190.0952	548.6817	415.3444	190.0170	595.3423
180	583.3406	190.0163	517.3045	387.2332	189.3435	587.2315
200	551.9715	189.8818	492.1313	377.1748	179.6745	567.1702

SETAR: Self-exciting threshold autoregressive, STAR: Smooth transition autoregressive

Table 11: Mean absolute percentage error of self-exciting threshold autoregressive and smooth transition autoregressive models fitted on PL (2): Xti= $0.7X_{ti-1}^2 - 0.6X_{ti-2} + e_t$ across the sample sizes

Sample size	SETAR (2,1)	SETAR (2,2)	SETAR (2,3)	STAR (2,1)	STAR (2,2)	STAR (2,3)
20	1340.7926	980.6726	1995.2816	1897.5615	18.0715	1255.9872
40	1320.4634	943.7916	1875.5932	1858.2432	17.6511	1243.0681
60	1318.8533	898.0138	1846.5414	1787.5018	17.2596	1223.0994
80	1298.3176	552.9945	1787.3855	1508.1065	16.7551	1204.0231
100	1257.6158	521.8885	1654.4346	1210.8366	16.5065	1169.0772
120	1244.9438	396.4515	1574.4774	1198.8048	16.1365	1172.0032
140	1216.2613	389.9954	1327.0113	1186.8262	16.0938	1156.7621
160	1183.6763	382.0065	1225.5437	1156.9183	15.9967	1135.0931
180	1165.5827	367.8916	1203.2573	1123.6213	15.6539	1124.8775
200	1153.0547	367.8731	1162.7307	1020.4163	15.3027	1090.0752

SETAR: Self-exciting threshold autoregressive, STAR: Smooth transition autoregressive

Table 12: Akaike information criteria of self-exciting threshold autoregressive and smooth transition autoregressive models fitted on PL (2): Xti= $0.7X_{ti-1}^2 - 0.6X_{ti-2} + e_t$ across the sample sizes

Sample size	SETAR (2,1)	SETAR (2,2)	SETAR (2,3)	STAR (2,1)	STAR (2,2)	STAR (2,3)
20	1340.7912	980.6720	1995.2842	1997.5645	118.0715	1010.0173
40	1150.4624	943.7914	1875.5967	1858.2445	117.6511	1009.1632
60	1188.8505	898.0165	1786.5432	1787.5050	117.2596	918.7866
80	1108.3111	552.9954	1687.3822	1508.1068	116.7551	918.7016
100	1097.6128	521.8885	1664.4313	1210.8318	116.5065	908.6115
120	1074.9462	396.4519	1574.4708	998.8074	116.1365	908.5015
140	1046.2612	389.9903	1527.0154	886.8266	116.0938	908.4335
160	1033.6735	382.0054	1485.5414	860.9141	115.9967	908.2214
180	1027.5814	367.8913	1453.2584	753.6258	115.6539	872.1825
200	1013.0575	367.8754	1362.7303	720.4166	115.3027	821.1074

SETAR: Self-exciting threshold autoregressive, STAR: Smooth transition autoregressive

based on AIC criteria. It was observed that SETAR (2, 1) and STAR (2, 1) of the models leading the other models at smaller and larger sample sizes, respectively, to fit the trigonometric form of autoregressive and therefore can be used to forecast for future values at different steps ahead for the respective sample sizes. SETAR (2, 3) performs far worse than the other models as the trends increases.

Table 7 shows the relative performance of the fitted models at different levels of sample size based on MSE criteria. It was observed that SETAR (2, 2) and STAR (2, 3) of the models exceed the other models at lower and higher sample sizes, respectively, to fit exponential form of autoregressive models and can be used to forecast for future values at different steps ahead. SETAR (2, 3) performs far worse than the other models as the trends increase higher than the other models.

Table 8 shows the relative performance of the fitted models at different levels of sample size based on AIC criteria. It was observed that SETAR (2,3) followed by STAR (2,3) models leading show the best performance model from the various sample sizes to fit exponential form of autoregressive models and therefore can be used to forecast for future values at different steps ahead. SETAR (2, 3) performs far worse than the other models as the trends increase higher than the other models.

Table 9 shows the relative performance of the fitted models at different levels of sample size based on AIC criteria. It was observed that SETAR (2, 2) and STAR (2, 3) of the models exceed the other models at lower and higher sample sizes, respectively, to fit exponential form of autoregressive models and therefore can be used to forecast for future values at different steps ahead. SETAR (2, 3) performs far worse than the other models as the trends increase higher than the other models.

Table 10 shows the relative performance of the fitted models at different levels of sample size based on AIC criteria. It was observed that STAR (2, 2) and SETAR (2, 2) models supersede the other models at lower and higher sample sizes, respectively, which show the best performance model to fit polynomial form of autoregressive models, and therefore, they can be used to forecast for future values for the respective sample sizes at different steps ahead. SETAR (2, 3) performs worse than the other models as the trends increase higher than the other models.

Table 11 shows the relative performance of the fitted models at different levels of sample size based on AIC criteria. It was observed that STAR (2, 2) and SETAR (2, 2) models supersede the other models at lower and higher sample sizes, respectively, which show the best performance model to fit polynomial form of autoregressive models, and therefore, they can be used to forecast for future values for the respective sample sizes at different steps ahead. SETAR (2, 3) performs worse than the other models as the trends increase higher than the other models.

Table 12 shows the relative performance of the fitted models at different levels of sample size based on AIC criteria. It was observed that STAR (2, 2) and SETAR (2, 2) models supersede the other models at lower and higher sample sizes, respectively, which show the best performance model to fit polynomial form of autoregressive models, and therefore, they can be used to forecast for future values for the respective sample sizes at different steps ahead. SETAR (2, 3) performs worse than the other models as the trends increase higher than the other models.[25]

CONCLUSION

In this study, comparative performance of the nonlinear models with non-stationarity features was carried out. It was concluded that the SETAR (2, 1) is the best model followed by SETAR (2, 2) to fit linear data, whereas SETAR (2, 2) forecasted better at different steps ahead. In fitting trigonometric nonlinear form of data, it can be seen that SETAR (2, 1) and STAR (2, 1) are known to be the best at lower and higher sample sizes, respectively, while in forecasting, STAR (2, 1) outperforms SETAR (2, 1). The SETAR (2, 2) and STAR (2, 3) are considered to be the best for an exponential and SETAR (2, 2) and STAR (2, 2) for a polynomial data. However, SETAR (2, 2) is shown to have the best forecast for both data.

REFERENCES

1. Akeyede I, Danjuma H, Bature TA. On consistency of tests for stationarity in autoregressive and moving average models of different orders. Am J Theor Appl Stat 2016a;5:146-53.
2. Akeyede I, Adeleke BL, Yahaya WB. Linear and nonlinear time series models selection for stationary data structure: Application to monthly Rainfall in Nigeria. J Phys Sci 2016b;21:1-13.
3. Tong H. On a threshold model. In: Chen CH, editor. Pattern Recognition and Signal Processing. Amsterdam: Sijthoff and Noordhoff; 1978. p. 101-41.
4. Akeyede I, Oyeyemi GM. On performance of some methods of detecting nonlinearity in stationary and non-stationary time series data. J Niger Stat Assoc 2016;28:95-105.
5. Tong H, Lim KS. Threshold auto-regression, limit cycles and cyclical data. J R Stat Soc 1980;B42:245-92.
6. Tong H. Threshold models in nonlinear time series analysis. New York: Springer-Verlag; 1983.
7. Aydin D, Guneri OI. Time series prediction using Hybridization of AR, SETAR and ARM models. Int J Appl Sci Technol 2015;5:87-96.
8. Clement MP, Smith J. Evaluating forecasts from SETAR models of exchange rates. J Int Money Finance 2001;20:133-48.

9. Clement MP, Smith J. A Monte Carlo study of the forecasting performance of empirical SETAR models. J Appl Econ 1997;5:31-25.
10. Keenan DM. A Tukey non-additivity type test for time series nonlinearity. Biometrika 1985;72:39-44.
11. Peel DA, Speight AE. The nonlinear time series properties of unemployment rates: Some further evidence. Appl Econ 1998;30:287-94.
12. Lundbergh S, Teräsvirta T. Forecasting with Smooth Transition Autoregressive Models. Working Paper Series in Economics and Finance, Stockholm School of Economics; 2002. p. 390.
13. Jorge V, Salvador T, Julian AF. STAR and ANN Models: Forecasting Performance on the Spanish Ibex-35 Stock Index Campus de Tafira. Spain: Tafira Baja. E-35017, Las Palmas; 2005.
14. Akeyede I, Yahaya WB, Adeleke BL. On forecast strength of some linear and nonlinear time series models for stationary data structure. Am J Math Stat 2015;5:163-77.
15. Fırat EH. SETAR (Self-exciting threshold autoregressive) non-linear currency modelling in EUR/USD, EUR/TRY and USD/TRY parities. Math Stat 2017;5:33-55.
16. Guidolin M, Stuart H, David MC, Sadayruki O. Non-linear predictability in stock and bond returns: When and where is it exploitable? Int J Forecast 2009;25:373-99.
17. Ismail MT, Isa Z. Modelling exchange rates using regime switching models. Sains Malaysiana 2006;35:55-62.
18. Samson BO, Awoeyo OO, Akinrefon AA, Yami AM. On the model selection of road accident data in Nigeria: A time series approach. Eur J Acad Essays 2015;2:33.
19. Sarantis N. Modeling non-linearities in real effective exchange rates. J Int Money Finance 1999;18:27-45.
20. Tayyab M, Tarar A, Riaz M. Threshold autoregressive (TAR) and momentum threshold autoregressive (MTAR) models specification. Res J Finance Acc 2012;3:23-31.
21. Terasvirta T, Dick VD, Marcelo MC. Linear Models, Smooth Transition Autoregressions and Neural Networks for Forecasting Macroeconomic Time Series: A Re-examination Texto Para Discussão, No. 485; 2005.
22. Tsay RS. Nonlinearity tests for time series. Biometrika 1986;73:461-6.
23. Umer M, Sevil T, Sevil G. Forecasting performance of smooth transition autoregressive (STAR) model on travel and leisure stock index. J Finance Data Sci 2018;4:90-100.
24. Zhou J. Smooth Transition Autoregressive Models a Study of the Industrial Production Index of Sweden June; 2010.
25. Tiao GC, Tsay RS. Some advances in nonlinear and adaptive modelling in time series. J Forecast 1994;13:109-14.