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REVIEW ARTICLE

A New Proof of the Pythagorean Theorem Using a Rhombus

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ABSTRACT
This short note presents a new proof of the Pythagorean Theorem using the concept of a rhombus.
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INTRODUCTION

In Euclidean geometry, the Pythagorean theorem is a fundamental relation between the three sides of a right-angled triangle. It states that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (Figure 1). If c is the hypotenuse of a right-angled triangle having a and b as the other two sides, then in mathematical form one can write the Pythagorean theorem as [1]

$$c^2 = a^2 + b^2$$

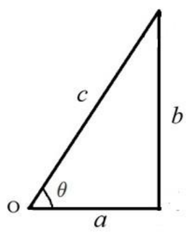

Figure 1. A right angled-triangle with sides a, b and c.

The theorem has hundreds of proofs and perhaps it has more known proofs than any other. E. S. Loomis' book The Pythagorean Proposition contains 370 proofs of the theorem [2]. The theorem has been proved by diverse methods which include both algebraic as well as geometric proofs. Some of the famous persons who proved the theorem include Euclid, U.S. President James A. Garfield, Albert Einstein etc. [3]. Two US teenagers have recently given a set of ten proofs of the theorem [4].

In the next section, the approach to obtain the Pythagorean relation will be obtained using the concept of a rhombus.

RHOMBUS APPROACH.

From Figure 1, the equation of the hypotenuse can be written as

$$y = mx = \frac{b}{a}x$$

where $m = {}^{b}/_{a}$ is the slope. Now construct a rhombus OBCD with hypotenuse as a side as shown in Figure 2.

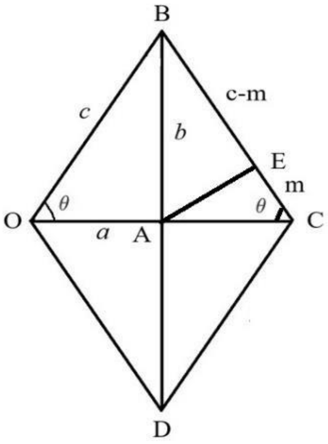

Figure 2. A rhombus generated from the initial right-angled triangle.

As area of one right-angled triangle having sides a and b is $\frac{1}{2}ab$, so the area of 4 triangles (rhombus) in terms of a and b is

$$A = 2ab \qquad (1)$$

In terms of the interior angle (2θ) and hypotenuse length c, area of the rhombus with side length c is

$$A = c^2 \sin 2\theta \qquad (2)$$

From Equations (1) and (2), we have

$$c^2 \sin 2\theta = 2ab \qquad (3)$$

Draw a perpendicular AE on BC such that CE = m and BE = c-m. Also, angle ABC = $(90^{\circ}-\theta)$.

Now, from the Figure 2

$$\frac{m}{AC} = \cos\theta$$

Or, $$m = a\cos\theta$$

and $$\frac{c-m}{b} = \cos\angle ABC = \cos(90 - \theta)$$

Or, $$c = m + b\sin\theta$$

$$c = a\cos\theta + b\sin\theta$$

Squaring the above equation, we get,

$$c^2 = a^2\cos^2\theta + b^2\sin^2\theta + ab\sin 2\theta$$

Using Equation (3) for $\sin 2\theta$ and the values for $\sin\theta$ and $\cos\theta$ from Figure 2 in the above equation, we get on simplification,

$$c^4 = a^4 + b^4 + 2a^2b^2 = (a^2 + b^2)^2$$

Or, $$c^2 = a^2 + b^2$$

Which is the Pythagorean relation. Mathematicians have given hundreds of proofs of the theorem using different geometries, at the same time rhombus geometry is also an option for the same.

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