AJMS
Asian Journal of Mathematical Sciences

RESEARCH ARTICLE

ON COMPUTATIONAL REVIEW OF THE BLOCK SCHAEFFER'S ITERATION FORMULA FOR STRONGLY PSEUDOCONTRACTIVE MAPS OF THE SYSTEM OF LINEAR EQUATIONS

*** Eziokwu, C. Emmanuel**

Department of Mathematics,
Michael Okpara University, Umudike, Abia State, Nigeria.

Corresponding Email: okereemm@yahoo.com

Received: 10-07-2025; Revised: 19-08-2025; Accepted: 05-09-2025

ABSTRACT

This work reviews the background concepts of the Block Schaeffer's fixed point iteration for- mula, states and proves its associated theorems before applying the method in the solution of a given system of linear equation, the aim of which is to computationally confirm that the traditional Block Schaeffer's iteration formula is strongly Pseudo-contractive on convergence. Again this research seeks to computationally reaffirm that the choice of any initial guess closer to the solution for an iteration formula converges faster to the solution. The obviousness of this is reflected in the main result highlighted in our computation which showed that a slight ad- justment in the initial guess becoming $\bar{x}^* = \bar{x}_0 \pm \bar{x}_0 10^{-1}$; $\bar{x}_0 \neq \bar{0}$ produces faster convergence automatically, no matter how close $\bar{x}_0$ is to the solution, x^*.

Keywords: Banach Space, System of Linear Equations, Fixed Point, Strong Pseudo-contraction, Iterative Method.

INTRODUCTION

Given a vector space, X a norm on X is a real valued function $\|.\|$ such that for all vector

$x \in X$

(i) $\|x\| \geq 0$, $\|x\| = 0$ if and only if $x = 0$
(ii) $\|\lambda x\| = |\lambda| \|x\|$, λ arbitrary scaler
(iii) $\|x + y\| \leq \|x\| + \|y\|$, $\forall\, x, y \in X$

A normed space [1, 5, 10, 15] is a vector space X with a norm $\|.\|$ defined on it. A complete normed space X is called a Banach space and in such X, every Cauchy sequence such that $\|x_n - x_m\| \to 0$ as $m, n \to \infty$ is a convergent sequence i.e. $\exists\, x \in X$ such that $\|x_n - x\| \to 0$ as $n \to \infty$.

1.1: Definition 1

(2, 6, 11, 16). Let $(X, \|.\|)$ be a Banach space and let $T : X \to X$ be Lipschitzian that is there exists a constant $K > 0$ where K is Lipschitz constant such that

$$\|T(x) - T(y)\| \leq K\|x - y\|, \ \forall \ x, y \in X$$

Then such a Lipschitz constant K less than 1($K < 1$) is called a contraction and expansive if

$K > 1$

Theorem 1. (BCMP,3, 7, 12, 17). Let $(X, \|.\|)$, $X \neq \emptyset$ be a Banach space and $T : X \to X$ a contraction on X. Then T has a unique fixed point

1.2: Definition 2

(4, 8, 9, 13, 14, 18, 19). Let X be a normed linear vector space. If $T : X \to X$ be a map such that $Tx = x$, $x \in X$. Then x is a fixed point of the set X provided

$$x_{n+1} = Tx_n; \ (n \geq 0) \tag{1}$$

Main results on the block schaeffer's iterative formula

A fixed point map is said to be strongly pseudo-contractive in a linear Banach space with the transformed matrix operator $Ax = B$ resulting in $x = Tx = Ax + B$ so that we have

$$\|(1 - r) I + rtA\| > 1$$

Whenever

$\|x_1 - x_2\| \ \| (1 - r)(x_1 - x_2) - rt (T(x_1) - T(x_2))\|$

$= \|(1 + r)(x_1 - x_2) - rtA(x_1 - x_2)\|$

$= \|(1 + r) I - rtA\| \| x_1 - x_2\| \| x_1 - x_2\|$

The above informs our definitions and results on the Block Schaeffer's block fixed point iteration method as presented below.

1.3: Definition 3. The traditional one step Banach's iteration method is called the k-block, r-point block formula if it assumes the form of a matrix of finite difference equation of the type

$$x_{i,n} = \sum_{j=1} AX_{i,n-j} + h \sum_{i=0} BF_{i,n-1} \tag{2}$$

where $X_n = (x_n, \ x_{n-1}, \ldots, \ x_{n-r+1})$ $F_n = (x_n, \ x_{n-1}, \ \ldots, \ x_{n-r+1})$ A's and B's properly chosen r x r (0, 1, 2 . . . , n) representing the nth block and r the proposed block size. In view of the above, the iteration procedure (1) and (2) are from henceforth rewritten in block form as

$$x_{i,\ n+1} = Tx_{i,n} \tag{3}$$

$$x_{i,n+1} = (1 - \lambda_{i,n})x_{i,n} + \lambda_{i,n}g(x_{i,n}); \tag{4}$$

$$0 \leq \lambda_{i,n} < 1, \quad \sum_{I=1} \lambda_{i,n} = 1 \text{ and } \sum_{I=1} \lambda$$

with the set of initial guess as $x^0_{i,n} = (x_{i,1}, , x_{i,2}, \ldots, x_{i,n})$ where the following results with top bars on them convey the same meaning as in this very definition

Theorem 2. (The Block Schaeffer's iterative method). Let the linear fixed point problem $A\bar{x} =$ B be well defined in the Banach space $(X, \|.\|)$ such that the transformed matrix A is diagonal dominant and

$$\bar{x} = T\bar{x} = A\bar{x} + B, \quad \bar{x} \in X$$

If T is a strongly pseudo contractive map, then for $\bar{x}_0 = (\bar{x}^0, x^0, \ldots, x^0)$ the iterative method

$\bar{x}_{n+1} = (1 - \lambda_n)\bar{x}_n + \lambda_n g(\bar{x}_n); 0 \leqslant \lambda_n < 1, \sum_{n=1} \lambda_n = 1,$ *and* $\sum_{n=1} \lambda_n < \infty$ *converges to a unique* fixed point.

Proof. We know that if $\bar{x}$ is a fixed point for F $(\bar{x})$, then

$$\bar{x} = F(\bar{x}) = A\bar{x} + \bar{B}$$

and if F $(\bar{x})$ is continuous and Lipschitzian, then

$$\|\bar{x}_1 - \bar{x}_2\| \leq \|(1 + \bar{r})(\bar{x}_1 - \bar{x}_2) - \bar{r}t(F(\bar{x}_1) - F(\bar{x}_2))\|$$

$$= \|(1 + \bar{r})(\bar{x}_1 - \bar{x}_2) - \bar{r}tA(\bar{x}_1 - \bar{x}_2)\|$$

$$= \|[(1 + \bar{r})I - \bar{r}tA](\bar{x}_1 - \bar{x}_2)\|$$

$$= \|(1 + \bar{r})I - \bar{r}tA\| \|(\bar{x}_1 - \bar{x}_2)\| \geq \|\bar{x}_1 - \bar{x}_2\|$$

If $\|(1 + \bar{r})I - \bar{r}tA\| > 1$, the map or the system of linear equations is strongly pseudo contractive then the Block Schaeffer's iterative method, $\bar{x}_{i,n+1} = (1 - \bar{\lambda}_{i,n})\bar{x}_{i,n} + \bar{\lambda}_{i,n}g(\bar{x}_{i,n})$; 0 $\bar{\lambda}_n < 1, \sum_{n=1}^{\infty} \bar{\lambda}_n = 1$, and $\sum_{n=1}^{\infty} \bar{\lambda}_n^2 < \infty$ becomes suitable for the solution of such system of linear equations, and we now show that $\bar{x}_{n+1}$ converges to a unique fixed point $\bar{x}^*$. First we know that the sequence $\{\bar{x}_n\}$ is Cauchy in $(X, \|.\|)$ and X is complete, hence $\{\bar{x}_n\}$ converges to a point in X as shown below. Let $\bar{x}_n \to \bar{x}^*$ as $n \to \infty$. Since F is a strongly pseudo-contraction it follows from the above, that $F_n(\bar{x}) \to F(\bar{x})$ as $n \to \infty$ but $F(\bar{x}_n) = \bar{x}_{n+1}$ so that

$$\bar{x}_{n+1} = F(\bar{x}_n) = F(\bar{x}^*)$$

and so hence, we prove that F has a unique fixed point in (X, ||·□). We assume that, suppose for contradiction there exists x¯1, ¯x2 ∈ X such that F (x¯1) = x¯1, F (x¯2) = x¯2 so that

$$\|\bar{x}_1 - \bar{x}_2\| \leq \|(1+\bar{r})(\bar{x}_1 - \bar{x}_2) - \bar{r}t(F(\bar{x}_1) - F(\bar{x}_2))\|$$

$$= \|(1+\bar{r})(\bar{x}_1 - \bar{x}_2) - \bar{r}tA(\bar{x}_1 - \bar{x}_2)\| = \|[(1+r)+\bar{r}tA](\bar{x}_1 - \bar{x}_2)\|$$
$$= \|(1+\bar{r})I - \bar{r}tA\|\,\|\bar{x}_1 - \bar{x}_2\| \geq \|\bar{x}_1 - \bar{x}_2\|$$

and

$$\|(1+\bar{r})I - \bar{r}tA\| > 1$$

which is strongly a pseudo-contraction and hence $\bar{x}^*_1 = \bar{x}^*_2$, meaning that

$$\bar{x}_{i,n+1} = 1 - \bar{\lambda}_n \;\; \bar{x}_{i,n} + \bar{\lambda}_n g\;(\bar{x}_{i,n});$$

$$0 \leq \bar{\lambda}_n < 1, \sum_{n=1}^{\infty} \bar{\lambda}_n = 1, \text{ and } \sum_{n=1}^{\infty} \bar{\lambda}_n^2 < \infty \text{ has a unique fixed point.}$$

Conclusively, it is sufficient to prove that if we have

$$\bar{x}_{i,n+1} = 1 - \bar{\lambda}_n \;\; \bar{x}_{i,n} + \bar{\lambda}_n g\;(\bar{x}_{i,n});0 \leq \bar{\lambda}_n < 1, \sum \bar{\lambda}_n = 1, \text{and} \sum \bar{\lambda}^2 < \infty \tag{5}$$

for the solution of the system

$$A\bar{x} = \bar{b}, x \in \mathbb{R}^n \tag{6}$$

which we transform to

$$\bar{x} = A\bar{x} + \bar{b} \tag{7}$$

and then define g (x¯) = Ax¯ + ¯b, so that there exists a constant a > 0 such that for every x¯

$$\bar{x}(I - f)\bar{x} \rightarrow cx\bar{x}$$

Then (6) as well as (7) has a unique solution x¯∗ and the iteration process generated from any x¯0 ∈ Xn by x¯i,n+1 = $\frac{1}{2}$ (x¯i,n + g (x¯i,n)) converges to x¯∗ provided that in g (x¯) = Ax¯ + ¯b

$$\sum_{i=1}\sum_{j=1} |\bar{a}_{ij}| \geqslant \bar{\mu} = 1,\;\; 1 \leqslant i \leqslant n,\;\; 1 \leqslant j \leqslant n$$

Hence the proof.

□

Remark: The above theorem is a reformulation of the following; Let the Block Schaeffer's iteration formula be such that g is strongly pseudo-contractive and

(I − g)

is accretive for $\bar{x}^{-t}A\bar{x} \rightarrow 0$ r implying that A is positive semi-definite for $\bar{x} \neq 0$, $\bar{x} \in H$, a Hilbert space or if A is positive definite, then $\bar{x}^{-t}A\bar{x} > 0$ for $\bar{x} \in H$ such that $\bar{x} \neq 0$.

Hence such an A can be strictly monotony or accretive only when $^{-\,-t}A\bar{x} \rightarrow C\bar{x}^t$ for some constant c > 0 provided g = (I − A) is strongly accretive.

Theorem 3. Let the system of linear equations

$$A\bar{x} = \bar{b} \; for \; \bar{x}, \quad b \in \mathbb{R}, \; A \in M_n(\mathbb{R}^n) \tag{8}$$

be given and the transformed system becomes

$$\bar{x} = A\bar{x} + B \tag{9}$$

if there exists a constant c > 0 such that ∀x¯

$$\bar{x}^+ (I - g)\bar{x} \rightarrow c\bar{x}^t\bar{x}$$

Then (8) as well as (9) above has unique fixed solution x¯∗ and the iteration process generated from any x¯0 ∈ Rn by the Block Schaeffer's iteration process

$$\bar{x}_{i,n+1} = 1 - \bar{\lambda}_n \;\; \bar{x}_{i,n} + \bar{\lambda}_n g(\bar{x}_{i,n}), n \geq 0$$

$$0 \leq \bar{\lambda}_n < 1, \sum_{n=1}^{\infty} \bar{\lambda}_n = 1, \; and \; \sum_{n=1}^{\infty} \bar{\lambda}_n^2 < \infty \; converges \; strongly \; to \; \bar{x}^* \;|$$

Proof. let $\bar{x} = g(\bar{x}) = A\bar{x} + B$ so that $H = \mathbb{R}^n$ and A an $n \times n$ matrix we show that

$$\|(1 - \bar{r})I - \bar{r}tA\| > 1$$

But,

$$\begin{aligned} &\|(1 + \bar{r})(\bar{x} + \bar{y}) - \bar{r}t(g(\bar{x}) - g(\bar{y}))\| \\ &= \|(1 - \bar{r})(\bar{x} + \bar{y}) - \bar{r}tA(\bar{x} - \bar{y})\| \\ &= \|[(1 - \bar{r})I - \bar{r}tA](\bar{x} + \bar{y})\| \;| \\ &= \|(1 - \bar{r})I - \bar{r}t\| \, \|\bar{x} - \bar{y}\| \geq \|\bar{x} - \bar{y}\| \end{aligned}$$

If $\|(1 + \bar{r})\,l - \bar{r}\,t\| > 1$ then the iteration process is strongly pseudo-contractive provided

$$0 \leq \bar{\lambda}_n < 1, \sum_{n=1}^{\infty} \bar{\lambda}_n = 1, \text{and} \sum_{n=1}^{\infty} \bar{\lambda}_n^2 < \infty$$

□

Theorem 4. Given the linear fixed point problem F = AX +B awhich is well defined in (X, ‖.‖) and the transformed matrix A, diagonal dominant with

$$\bar{x} = F\bar{x} = A\bar{x} + B, \quad \bar{x} \in X$$

Then for F a strongly pseudo-contractive iterative method of solution $\bar{x}_{i,n+1} = (1-\lambda_n)\bar{x}_{i,n} + \lambda_n g(\bar{x}_{i,n})$; $0 \leq \lambda_n < 1$, $\sum_{n=1}^{\infty} \lambda_n = 1$, and $\sum_{n=1}^{\infty} \lambda_n < \infty$ There exists a better initial guess

$$\bar{x}^*_0 = \bar{x}_0 \pm \bar{x}_0 \, 10^{-1} \; ; \; \bar{x}_0 \neq \bar{0}$$

which produces a faster convergent solution than the original initial guess $\bar{x}_0 = (x_1^0, x_2^0, \ldots, x_n^0)$ does no matter how close is the initial guess to the solution.

Proof. Let

$$F(\bar{x}) = A\bar{x} - B = 0$$

If $\bar{x}_1$ is the fixed point of $F(\bar{x}) = A\bar{x} - B$, then for $\bar{x}^0 = (x_1^0, x_2^0, \ldots, x_n^0)$

$$\bar{x}_1 = F(\bar{x}_0) = \bar{x}$$

Also if $\bar{x}_{n-m} = (x_1^{n-m}, x_2^{n-m}, \ldots, x_n^{n-m})$ is a fixed point for $F(\bar{x}) = A\bar{x} - B = 0$ then

$$\bar{x}_{n-m} = F(\bar{x}_{n-m-1}) = \bar{x}$$

By induction we assume that for $\bar{x}^n = (x_1^n, x_2^n, \ldots, x_n^n)$,

$$F(\bar{x}) = A\bar{x} - B = 0$$

So that $\bar{x}_n = F(\bar{x}_{n-1}) = \bar{x}$ is again a fixed point. Hence,
for $\bar{x}_{n+1} = (x_1^{n+1}, x_2^{n+1}, \ldots, x_n^{n})$

$$\bar{x}_{n+1} = F(\bar{x}) = \bar{x}$$

also a fixed point for

$$F(\bar{x}) = A\bar{x} - B = 0$$

But we know that fixed points for any given problem is unique, hence,

$$\bar{x}_1 - \bar{x}_{n-m} = \bar{x}_n = \bar{x}_{n+1}$$

But by iteration $\bar{x}_{i,1}$ is far behind $\bar{x}_{i,n-m}$ far behind $\bar{x}_{i,n}$ behind $\bar{x}_{i,n+1}$. Therefore,

$$\bar{x}_{i,1} \subset \ldots \subset \bar{x}_{i,n-m} \subset \ldots \subset \bar{x}_{i,n} \subset \ldots \subset \bar{x}_{i,n+1}$$

Where $\bar{x}_{i,1} \rightarrow \bar{x}$, $\bar{x}_{i,n-m} \rightarrow \bar{x}$, $\bar{x}_{i,n} \rightarrow \bar{x}$ and $\bar{x}_{i,n+1} \rightarrow \bar{x}$ is unique implies that fixed point $\bar{x}$ is unique. If $\bar{x}_{i,n-m} \rightarrow \bar{x}$ with the initial guess

$$\bar{x}^*_0 = \bar{x}_0 \pm \bar{x}_0 \; 10^{-1}$$

and

$$\bar{x}_{n-m} \subset \bar{x}_{n+1}$$

Then $\bar{x}_{i,n-m} = \bar{x}$ with $\bar{x}^*_0 = \bar{x}_0 \pm \bar{x}_0 \; 10^{-1}$; $\bar{x}_0 \neq \bar{0}$ has a faster convergence than $\bar{x}_{i,n+1} = \bar{x}$ with $\bar{x}_{i,0} = (x^0_1, x^0_2, \ldots, x^0_n)$. Hence, the proof. □

COMPUTATIONAL EXPERIMENT

3.1 Experiment

Illustrate the claim of the above result by using the Traditional Block Schaeffer's iteration formula on the given system of linear equations below with initial guess (0, 0, 0).

$$3x + 5y + 6z = 8$$
$$x + 7y + 2z = 18$$
$$2x - 3y + 3z = 1$$

Solution. Observe that the system is not diagonal dominant but can be made so by applying the rule that will ensure that the law of maximum norm is satisfied. If that is done, then the above system of linear equation transformed into the following system of linear equations.

$$4x + 2y - 3z = 9$$
$$x + 7y - 2z = 18$$
$$2x + 5y - 6z = 8$$

Hence the transformed system of linear equations is rewritten as below so that by applying the Traditional Block Schaeffer's fixed point iteration formula; xn+1 $= (1 - \lambda_n)\; x_n + \lambda_n g\; (x_n) : 0 < \lambda_n < 1$ the computation of the iteration process is initiated as below

$$x^{(n)}_1 = \left[2.25 - 0.5x^{(n)}_2 - 0.75x^{(n-1)}_3\right]\lambda_n$$

$$x^{(n)}_2 = \left[2.571428571 - 0.142857142x^{(n)}_1 - 2.85714285x^{(n-1)}_3\right]\lambda_n$$

$$x^{(n)}_3 = \left[-1.333333333 + 0.333333333x^{(n)}_1 - 0.833333333x^{(n-1)}_2\right]\lambda_n$$

And the following tables of results are generated by applying the python programming computer source codes of below.

Table 1: Computational Result Output for the Traditional Block Schaeffer's Fixed Point Iteration Formula with the Initial Guess (0, 0, 0)

```
D:\project works\Numerical Methods>python schaeffer.py
Enter number equations {3x3}: 3
> 4x+2y-3z=9
> 1x+7y+2z=18
> 2x+5y-6z=8
```

n	λn	1-λn	Xn1	Xn2	Xn3
0	0.0000000	1.00000000	0.0	0.0	0.0
1	0.001	0.999	0.00225	0.00257	-0.00133
2	0.002	0.998	0.0067455	0.00770486	-0.00398734
3	0.003	0.997	0.0134752635	0.01539174542	-0.00796537798
4	0.004	0.996	0.022421362446	0.025610178438	-0.013253516468
5	0.005	0.995	0.033559255634	0.038332127546	-0.019837248886
6	0.006	0.994	0.0468579001	0.053522134781	-0.027698225392
7	0.007	0.993	0.062279894799	0.071137479837	-0.036814337815
8	0.008	0.992	0.079781655641	0.091128379999	-0.047159823112
9	0.009	0.991	0.09931362074	0.113438224579	-0.058705384704
10	0.01	0.99	0.120820484533	0.138003842333	-0.071418330857
11	0.011	0.989	0.144241459203	0.164755800067	-0.085262729218
12	0.012	0.988	0.169510561692	0.193618730466	-0.100199576467
13	0.013	0.987	0.19655692439	0.22451168697	-0.116186981973
14	0.014	0.986	0.225305127449	0.257348523353	-0.133180364225
15	0.015	0.985	0.255675550537	0.292038295503	-0.151132658762
16	0.016	0.984	0.287584741729	0.328485682774	-0.169994536222
17	0.017	0.983	0.320945801119	0.366591426167	-0.189714629106
18	0.018	0.982	0.355668776699	0.406252780496	-0.210239765782
19	0.019	0.981	0.391661069942	0.447363977667	-0.231515210232
20	0.02	0.98	0.428827848543	0.489816698114	-0.253484906028
21	0.021	0.979	0.467072463724	0.533500547453	-0.276091723001
22	0.022	0.978	0.506296869522	0.578303535409	-0.299277705095
23	0.023	0.977	0.546402041523	0.624112554095	-0.322984317878
24	0.024	0.976	0.587288392526	0.670813852796	-0.347152694249
25	0.025	0.975	0.628856182713	0.718293506477	-0.371723876893
26	0.026	0.974	0.671005921962	0.766437875308	-0.396639056093
27	0.027	0.973	0.713638762069	0.815134052675	-0.421839801579
28	0.028	0.972	0.756656876731	0.8642702992	-0.447268287135
29	0.029	0.971	0.799963827306	0.913736460523	-0.472867506808
30	0.03	0.97	0.843464912487	0.963424366707	-0.498581481603
31	0.031	0.969	0.8870675002	1.01322821134	-0.524355455674
32	0.032	0.968	0.930681340194	1.063044908577	-0.550136081092
33	0.033	0.967	0.974218855967	1.112774426594	-0.575871590416
34	0.034	0.966	1.017595414864	1.162320096089	-0.601511956342
35	0.035	0.965	1.060729575344	1.211588892726	-0.62700903787

36	0.036	0.964	1.103543310632	1.260491692588	-0.652316712507
37	0.037	0.963	1.145962208138	1.308943499962	-0.677390994144
38	0.038	0.962	1.187915644229	1.356863646964	-0.702190136367
39	0.039	0.961	1.229336934104	1.404175964732	-0.726674721048
40	0.04	0.96	1.27016345674	1.450808926143	-0.750807732206
41	0.041	0.959	1.310336755014	1.496695760171	-0.774554615186
42	0.042	0.958	1.349802611303	1.541774538244	-0.797883321348
43	0.043	0.957	1.388511099017	1.585988233099	-0.82076433853
44	0.044	0.956	1.42641661066	1.629284750843	-0.843170707635
45	0.045	0.955	1.463477863181	1.671616937055	-0.865078025791
46	0.046	0.954	1.499657881474	1.712942557951	-0.886464436605
47	0.047	0.953	1.534923961045	1.753224257727	-0.907310608084
48	0.048	0.952	1.569247610915	1.792429493356	-0.927599698896
49	0.049	0.951	1.60260447798	1.830530448182	-0.94731731365
50	0.05	0.95	1.634974254081	1.867503925772	-0.966451447968
51	0.051	0.949	1.666340567123	1.903331225558	-0.984992424122
52	0.052	0.948	1.696690857632	1.937998001829	-1.002932818067
53	0.053	0.947	1.726016242178	1.971494107732	-1.02026737871
54	0.054	0.946	1.7543113651	2.003813425915	-1.036992940259
55	0.055	0.945	1.78157424002	2.034953687489	-1.053108328545
56	0.056	0.944	1.807806082579	2.06491628099	-1.068614262147
57	0.057	0.943	1.833011135872	2.093706052973	-1.083513249204
58	0.058	0.942	1.857196489991	2.121331101901	-1.09780948075
59	0.059	0.941	1.880371897082	2.147802566889	-1.111508721386
60	0.06	0.94	1.902549583257	2.173134412876	-1.124618198103
61	0.061	0.939	1.923744058678	2.19734321369	-1.137146488019
62	0.062	0.938	1.94397192704	2.220447934441	-1.149103405761
63	0.063	0.937	1.963251695637	2.242469714572	-1.160499891198
64	0.064	0.936	1.981603587116	2.263431652839	-1.171347898162
65	0.065	0.935	1.999049353953	2.283358595404	-1.181660284781
66	0.066	0.934	2.015612096592	2.302276928108	-1.191450705986
67	0.067	0.933	2.031316086121	2.320214373925	-1.200733508685
68	0.068	0.932	2.046186592264	2.337199796498	-1.209523630094
69	0.069	0.931	2.060249717398	2.353263010539	-1.217836499618
70	0.07	0.93	2.07353223718	2.368434599802	-1.225687944644
71	0.071	0.929	2.086061448341	2.382745743216	-1.233094100575
72	0.072	0.928	2.09786502406	2.396228049704	-1.240071325333
73	0.073	0.927	2.108970877304	2.408913402076	-1.246636118584
74	0.074	0.926	2.119407032383	2.420833810322	-1.252805045809
75	0.075	0.925	2.129201504954	2.432021274548	-1.258594667373
76	0.076	0.924	2.138382190578	2.442507657682	-1.264021472653
77	0.077	0.923	2.146976761903	2.452324568041	-1.269101819258
78	0.078	0.922	2.155012574475	2.461503251734	-1.273851877356

79	0.079	0.921	2.162516581091	2.470074494847	-1.278287579045
80	0.08	0.92	2.169515254604	2.478068535259	-1.282424572722
81	0.081	0.919	2.176034518981	2.485514983903	-1.286278182331
82	0.082	0.918	2.182099688425	2.492442755223	-1.28986337138
83	0.083	0.917	2.187735414285	2.498880006539	-1.293194711555
84	0.084	0.916	2.192965639485	2.50485408599	-1.296286355785
85	0.085	0.915	2.197813560129	2.510391488681	-1.299152015543
86	0.086	0.914	2.202301593958	2.515517820654	-1.301804942206
87	0.087	0.913	2.206451355284	2.520257770257	-1.304257912234
88	0.088	0.912	2.210283636019	2.524635086475	-1.306523215958
89	0.089	0.911	2.213818392413	2.528672563779	-1.308612649738
90	0.09	0.91	2.217074737096	2.532392033038	-1.310537511261
91	0.091	0.909	2.22007093602	2.535814358032	-1.312308597736
92	0.092	0.908	2.222824409906	2.538959437093	-1.313936206745
93	0.093	0.907	2.225351739785	2.541846209443	-1.315430139517
94	0.094	0.906	2.227668676245	2.544492665756	-1.316799706403
95	0.095	0.905	2.229790152002	2.546915862509	-1.318053734294
96	0.096	0.904	2.23173029741	2.549131939708	-1.319200575802
97	0.097	0.903	2.233502458561	2.551156141556	-1.320248119949
98	0.098	0.902	2.235119217622	2.553002839684	-1.321203804194
99	0.099	0.901	2.236592415077	2.554685558555	-1.322074627579
100	0.1	0.9	2.23793317357	2.5562170027	-1.322867164821
101	0.101	0.899	2.239151923039	2.557609085427	-1.323587581174
102	0.102	0.898	2.240258426889	2.558872958713	-1.324241647894
103	0.103	0.897	2.24126180892	2.560019043966	-1.324834758161
104	0.104	0.896	2.242170580792	2.561057063393	-1.325371943313
105	0.105	0.895	2.242992669809	2.561996071737	-1.325857889265
106	0.106	0.894	2.243735446809	2.562844488133	-1.326296953003
107	0.107	0.893	2.244405754	2.563610127903	-1.326693179031
108	0.108	0.892	2.245009932568	2.564300234089	-1.327050315696
109	0.109	0.891	2.245553849918	2.564921508574	-1.327371831285
110	0.11	0.89	2.246042926427	2.56548014263	-1.327660929844
111	0.111	0.889	2.246482161594	2.565981846798	-1.327920566631
112	0.112	0.888	2.246876159495	2.566431879957	-1.328153463168
113	0.113	0.887	2.247229153472	2.566835077522	-1.32836212183
114	0.114	0.886	2.247545029977	2.567195878684	-1.328548839942
115	0.115	0.885	2.247827351529	2.567518352636	-1.328715723348
116	0.116	0.884	2.248079378752	2.56780622373	-1.32886469944
117	0.117	0.883	2.248304091438	2.568062895554	-1.328997529606
118	0.118	0.882	2.248504208648	2.568291473878	-1.329115821112
119	0.119	0.881	2.248682207819	2.568494788487	-1.3292210384
120	0.12	0.88	2.248840342881	2.568675413868	-1.329314513792
121	0.121	0.879	2.248980661392	2.56883568879	-1.329397457623

122	0.122	0.878	2.249105020702	2.568977734758	-1.329470967793
123	0.123	0.877	2.249215103156	2.569103473383	-1.329536038754
124	0.124	0.876	2.249312430365	2.569214642683	-1.329593569949
125	0.125	0.875	2.249398376569	2.569312812348	-1.329644373705
126	0.126	0.874	2.249474181121	2.569399397992	-1.329689182618
127	0.127	0.873	2.249540960119	2.569475674447	-1.329728656426
128	0.128	0.872	2.249599717224	2.569542788118	-1.329763388403
129	0.129	0.871	2.249651353702	2.569601768451	-1.329793911299
130	0.13	0.87	2.249696677721	2.569653538552	-1.32982070283
131	0.131	0.869	2.249736412939	2.569698925002	-1.32984419076
132	0.132	0.868	2.249771206431	2.569738666901	-1.329864757579
133	0.133	0.867	2.249801635976	2.569773424204	-1.329882744821
134	0.134	0.866	2.249828216755	2.56980378536	-1.329898457015
135	0.135	0.865	2.249851407493	2.569830274337	-1.329912165318
136	0.136	0.864	2.249871616074	2.569853357027	-1.329924110835
137	0.137	0.863	2.249889204672	2.569873447114	-1.329934507651
138	0.138	0.862	2.249904494427	2.569890911412	-1.329943545595
139	0.139	0.861	2.249917769702	2.569906074726	-1.329951392757
140	0.14	0.86	2.249929281944	2.569919224264	-1.329958197771
141	0.141	0.859	2.24993925319	2.569930613643	-1.329964091885
142	0.142	0.858	2.249947879237	2.569940466506	-1.329969190838
143	0.143	0.857	2.249955332506	2.569948979795	-1.329973596548
144	0.144	0.856	2.249961764625	2.569956326705	-1.329977398645
145	0.145	0.855	2.249967308754	2.569962659333	-1.329980675841
146	0.146	0.854	2.249972081676	2.56996811107	-1.329983497169
147	0.147	0.853	2.24997618567	2.569972798743	-1.329985923085
148	0.148	0.852	2.249979710191	2.569976824529	-1.329988006468
149	0.149	0.851	2.249982733372	2.569980277674	-1.329989793504
150	0.15	0.85	2.249985323366	2.569983236023	-1.329991324479
151	0.151	0.849	2.249987539538	2.569985767384	-1.329992634483
152	0.152	0.848	2.249989433528	2.569987930741	-1.329993754041
153	0.153	0.847	2.249991050198	2.569989777338	-1.329994709673
154	0.154	0.846	2.249992428468	2.569991351628	-1.329995524383
155	0.155	0.845	2.249993602055	2.569992692125	-1.329996218104
156	0.156	0.844	2.249994600135	2.569993832154	-1.32999680808
157	0.157	0.843	2.249995447914	2.569994800506	-1.329997309211
158	0.158	0.842	2.249996167143	2.569995622026	-1.329997734356
159	0.159	0.841	2.249996776567	2.569996318124	-1.329998094593
160	0.16	0.84	2.249997292317	2.569996907224	-1.329998399458
161	0.161	0.839	2.249997728254	2.569997405161	-1.329998657146
162	0.162	0.838	2.249998096277	2.569997825525	-1.329998874688
163	0.163	0.837	2.249998406584	2.569998179964	-1.329999058114
164	0.164	0.836	2.249998667904	2.56999847845	-1.329999212583

165	0.165	0.835	2.2499988877	2.569998729506	-1.329999342507
166	0.166	0.834	2.249999072342	2.569998940408	-1.329999451651
167	0.167	0.833	2.24999922726	2.56999911736	-1.329999543225
168	0.168	0.832	2.249999357081	2.569999265643	-1.329999619963
169	0.169	0.831	2.249999465734	2.56999938975	-1.329999684189
170	0.17	0.83	2.249999556559	2.569999493492	-1.329999737877
171	0.171	0.829	2.249999632388	2.569999580105	-1.3299997827
172	0.172	0.828	2.249999695617	2.569999652327	-1.329999820076
173	0.173	0.827	2.249999748275	2.569999712474	-1.329999851203
174	0.174	0.826	2.249999792075	2.569999762504	-1.329999877093
175	0.175	0.825	2.249999828462	2.569999804066	-1.329999898602
176	0.176	0.824	2.249999858653	2.56999983855	-1.329999916448
177	0.177	0.823	2.249999883671	2.569999867127	-1.329999931237
178	0.178	0.822	2.249999904378	2.569999890778	-1.329999943477
179	0.179	0.821	2.249999921494	2.569999910329	-1.329999953594
180	0.18	0.82	2.249999935625	2.56999992647	-1.329999961947
181	0.181	0.819	2.249999947277	2.569999939779	-1.329999968835
182	0.182	0.818	2.249999956873	2.569999950739	-1.329999974507
183	0.183	0.817	2.249999964765	2.569999959754	-1.329999979172
184	0.184	0.816	2.249999971248	2.569999967159	-1.329999983004
185	0.185	0.815	2.249999976567	2.569999973235	-1.329999986149
186	0.186	0.814	2.249999980926	2.569999978213	-1.329999988725
187	0.187	0.813	2.249999984493	2.569999982287	-1.329999990833
188	0.188	0.812	2.249999987408	2.569999985617	-1.329999992557
189	0.189	0.811	2.249999989788	2.569999988336	-1.329999993964
190	0.19	0.81	2.249999991728	2.569999990552	-1.32999999511
191	0.191	0.809	2.249999993308	2.569999992356	-1.329999996044
192	0.192	0.808	2.249999994593	2.569999993824	-1.329999996804
193	0.193	0.807	2.249999995637	2.569999995016	-1.329999997421
194	0.194	0.806	2.249999996483	2.569999995983	-1.329999997921
195	0.195	0.805	2.249999997169	2.569999996766	-1.329999998326
196	0.196	0.804	2.249999997724	2.5699999974	-1.329999998654
197	0.197	0.803	2.249999998172	2.569999997912	-1.32999999892
198	0.198	0.802	2.249999998534	2.569999998326	-1.329999999133
199	0.199	0.801	2.249999998826	2.569999998659	-1.329999999306
200	0.2	0.8	2.249999999061	2.569999998927	-1.329999999445
201	0.201	0.799	2.249999999249	2.569999999143	-1.329999999556
202	0.202	0.798	2.249999999401	2.569999999316	-1.329999999646
203	0.203	0.797	2.249999999523	2.569999999455	-1.329999999718
204	0.204	0.796	2.24999999962	2.569999999566	-1.329999999775
205	0.205	0.795	2.249999999698	2.569999999655	-1.329999999821
206	0.206	0.794	2.24999999976	2.569999999726	-1.329999999858
207	0.207	0.793	2.24999999981	2.569999999783	-1.329999999888
208	0.208	0.792	2.249999999849	2.569999999828	-1.329999999911

208	0.208	0.792	2.249999999849	2.569999999828	-1.329999999911
209	0.209	0.791	2.249999999881	2.569999999864	-1.32999999993
210	0.21	0.79	2.249999999906	2.569999999892	-1.329999999944
211	0.211	0.789	2.249999999926	2.569999999915	-1.329999999956
212	0.212	0.788	2.249999999941	2.569999999933	-1.329999999965
213	0.213	0.787	2.249999999954	2.569999999947	-1.329999999973
214	0.214	0.786	2.249999999964	2.569999999959	-1.329999999979
215	0.215	0.785	2.249999999972	2.569999999968	-1.329999999983
216	0.216	0.784	2.249999999978	2.569999999975	-1.329999999987
217	0.217	0.783	2.249999999983	2.56999999998	-1.32999999999
218	0.218	0.782	2.249999999986	2.569999999984	-1.329999999992
219	0.219	0.781	2.249999999989	2.569999999988	-1.329999999994
220	0.22	0.78	2.249999999992	2.569999999991	-1.329999999995
221	0.221	0.779	2.249999999994	2.569999999993	-1.329999999996
222	0.222	0.778	2.249999999995	2.569999999994	-1.329999999997
223	0.223	0.777	2.249999999996	2.569999999996	-1.329999999998
224	0.224	0.776	2.249999999997	2.569999999997	-1.329999999998
225	0.225	0.775	2.249999999998	2.569999999997	-1.329999999999
226	0.226	0.774	2.249999999998	2.569999999998	-1.329999999999
227	0.227	0.773	2.249999999999	2.569999999998	-1.329999999999
228	0.228	0.772	2.249999999999	2.569999999999	-1.329999999999
229	0.229	0.771	2.249999999999	2.569999999999	-1.33

Converge at num: 230

Now, we see and conclude that the above given system of equations when solved with the traditional Block Schaeffer's iteration formula converges at
$\bar{x}_{n+1} = \bar{x}_{230} = \bar{x}^* =$ 2.249999999999, 2.569999999999, −1.330000000000). but that the convergence becomes faster if the initial guess becomes re-adjusted by $\bar{x}_0^{A,B} = \bar{x}_0 \pm 10^{-1}$.

REFERENCES

1. Altman, M., "Contractors and Contractor Directions Theory and Applications". Mercel Dekker, Inc. New York and Basel (2010)

2. Argyros, I., "Approximate Solution of Operator Equations with Applications" World Sci- entific Publishers, New Jersey (2008)

3. Blasius, D., "On the application of the Krasnosalskii's method in the solution of systems of linear equations" Journal of Numerical Analysis Vol 8(3) 12-26 (2012)

4. Carnahan, B., "Applied Numerical Methods". John Wiley and Sons, Inc. New York (1984)

5. Chidume, C.E. "Applicable functional Analysis (Fundamental Theorems and Applica- tions)". ICTP, Trieste (1996)

6. Chidume, C.E. Picard's iterative method of solution for systems of linear equations. International Journal of Mathematical Analysis and Optimization, 2:11-20 (2017)

7. Chika, M. Krasnoselskii's fixed point iterative method for linear system of equations. Journal of application and applied mathematics, 8:33-44 (2014)

8. Chika M.S. "Advanced Functional Analysis, a Lecture Note" Nnamdi Azikiwe University Awka, Nigeria5:54-78 (2004)

9. Cotan, V., ”Krasnosalskii's iterative method of solution and the system of linear equa- tions.” Journal Mathematical Analysis, Vol 1 (2008)

10. Eziokwu, C.E. ”Finer Linear and Non Linear Newton's Numeric Topological Fixed Point Iterative Method and their Convergence Analysis in the Solution of System of Equations” A Ph.D Desertation yet undefined (2017)

11. Eziokwu, E. C. “On The Topological Picard's Fixed Point Iterative Algorithm For The Simplex Method Of Optimization”, Journal of the Nigerian Mathematical Society, Vol. 38, Issue 3, pp. 491-513,(2019)

12. Eziokwu, C. Emmanuel, Akabuike, Nkiruka and Agwu, Emeka, “On Application of the Fixed-Point Theorem to the Solution of OrdinaryDifferential Equations” Asian Journal of Mathematical Sciences, Vol. 2017, issue 2. (2019)

13. Eziokwu, C. Emmanue and Moore, S. Chika. On the Seidel's Method, a Stronger Con- traction Fixed Point Iterative Method of Solution for Systems of Linear Equations, Asian Journal of Mathematical Sciences, Vol. 2017, issue 2. (2017)

14. Kreyzig, E.”Introductory Functional Analysis with Applications” John Wiley and Sons New York (1978)

15. Nayyar, G. ”Numerical Methods (A Programming Approach)”. S.K Kataria and Sons, New Delhi, India (2009)

16. Osilike, M.O. Fixed point iterative methods of solution for systems of Linear equations.

17. Journal of Numerical Analysis, 4:20-38 (2016)

18. Friegyes, R. and Nagy, S.Z. ”A Functional Analysis”. Dover Publications, Inc. New York (1990)

19. Patrick, E.,”Iterative methods and the system of linear equations, a computational ap- proach.” Journal of computational mathematics. Vol 4(6)30-38. (2000)

20. William, F.T. ”Introduction to Real Analysis”. San Antonio IX USA Edited by National Mathematical Centre, Abuja, Nigeria (2012)